Name ____ Date ________________

1. Jacob lives on a street that runs east and west. The grocery store is to the east and the post office is to the west of his house. Both are on the same street as his house. Answer the questions below about the following story:

   At 1:00 p.m., Jacob hops in his car and drives at a constant speed of 25 mph for 6 minutes to the post office. After 10 minutes at the post office, he realizes he is late and drives at a constant speed of 30 mph to the grocery store, arriving at 1:28 p.m. He then spends 20 minutes buying groceries.

   a. Draw a graph that shows the distance Jacob's car is from his house with respect to time. Remember to label your axes with the units you chose and any important points (home, post office, grocery store).

b. On the way to the grocery store, Jacob looks down at his watch and notes the time as he passes his house. What time is it when he passes his house? Explain how you found your answer.

c. If he drives directly back to his house after the grocery story, what was the total distance he traveled to complete his errands? Show how you found your answer.

2. Jason is collecting data on the rate of water usage in the tallest skyscraper in the world during a typical day. The skyscraper contains both apartments and businesses. The electronic water meter for the building displays the total amount of water used in liters. At noon, Jason looks at the water meter and notes that the digit in the **ones** place on the water meter display changes too rapidly to read the digit and that the digit in the **tens** place changes every second or so.

   a. Estimate the total number of liters used in the building during one 24-hour day. Take into account the time of day when he made his observation. (Hint: Will water be used at the same rate at 2:00 a.m. as at noon?) Explain how you arrived at your estimate.

   b. To what level of accuracy can Jason reasonably report a measurement if he takes it at precisely 12:00 p.m.? Explain your answer.

   c. The meter will be checked at regular time spans (for example, every minute, every 10 minutes, and every hour). What is the minimum (or smallest) number of checks needed in a 24-hour period to create a reasonably accurate graph of the water usage **rate** with respect to time? (For example, 24 checks would mean checking the meter every hour; 48 checks would mean checking the meter every half hour.) Defend your choice by describing how the water usage rate might change during the day and how your choice could capture that change.

3. A publishing company orders black and blue ink in bulk for its two-color printing press. To keep things simple with its ink supplier, each time it places an order for blue ink, it buys B gallons, and each time it places an order for black ink, it buys K gallons. Over a one-month period, the company places m orders of blue ink and n orders of black ink.

   a. What quantities could the following expressions represent in terms of the problem context?

   $m + n$

   $mB + nK$

   $\frac{mB+nK}{m+n}$

   b. The company placed twice as many orders for black ink than for blue ink in January. Give interpretations for the following expressions in terms of the orders placed in January,

   $$\frac{m}{m+n} \quad \text{and} \quad \frac{n}{m+n},$$

   and explain which expression must be greater using those interpretations.

4. Sam says that he knows a clever set of steps to rewrite the expression

$$(x + 3)(3x + 8) - 3x(x + 3)$$

as a sum of two terms where the steps do not involve multiplying the linear factors first and then collecting like terms. Rewrite the expression as a sum of two terms (where one term is a number and the other is a product of a coefficient and variable) using Sam's steps if you can.

5. Using only the addition and multiplication operations with the numbers 1, 2, 3, and 4 each exactly once, it is possible to build a numeric expression (with parentheses to show the order used to build the expression) that evaluates to 21. For example, $1 + ((2 + 3) \cdot 4)$ is one such expression.

   a. Build two more numeric expressions that evaluate to 21 using the criteria above. Both must be different from the example given.

b. In both of your expressions, replace 1 with a, 2 with b, 3 with c, and 4 with d to get two algebraic expressions. For example, $a + \left((b + c) \cdot d\right)$ shows the replacements for the example given.

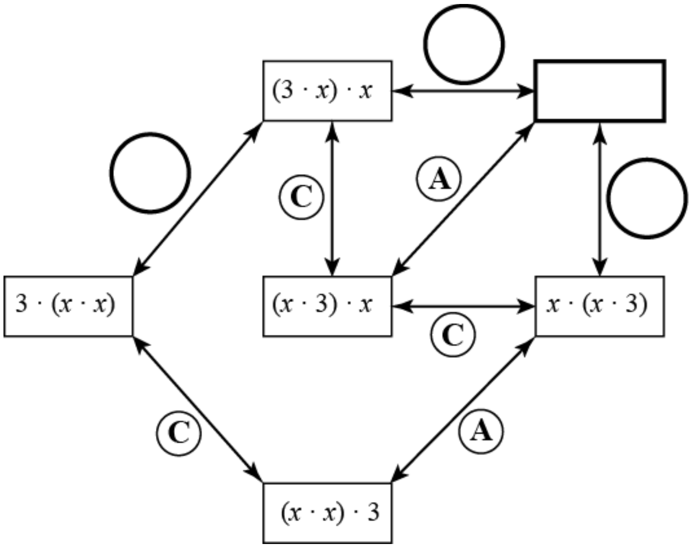
Are your algebraic expressions equivalent? Circle: Yes No

- If they are equivalent, prove that they are using the properties of operations.
- If not, provide **two** examples:

  (1) Find four different numbers (other than $0, 1, 2, 3, 4$) that when substituted for a, b, c, and d into each expression, the expressions evaluate to **different numbers**, and

  (2) Find four different, nonzero numbers that when substituted into each expression, the expressions evaluate to the **same number**.

6. The diagram below, when completed, shows all possible ways to build equivalent expressions of $3x^2$ using multiplication. The equivalent expressions are connected by labeled segments stating which property of operations, **A** for associative property and **C** for commutative property, justifies why the two expressions are equivalent. Answer the following questions about $3x^2$ and the diagram.



a. Fill in the empty circles with **A** or **C** and the empty rectangle with the missing expression to complete the diagram.

b. Using the diagram above to help guide you, give *two different* proofs that $(x \cdot x) \cdot 3 = (3 \cdot x) \cdot x$.

7. Ahmed learned: “To multiply a whole number by ten, just place a zero at the end of the number.” For example, 2813×10, he says, is $28{,}130$. He doesn't understand why this rule is true.

   a. What is the product of the polynomial $2x^3 + 8x^2 + x + 3$ times the polynomial x?

   b. Use part (a) as a hint. Explain why the rule Ahmed learned is true.

8.

   a. Find the following products:

      i. $(x - 1)(x + 1)$

      ii. $(x - 1)(x^2 + x + 1)$

      iii. $(x - 1)(x^3 + x^2 + x + 1)$

      iv. $(x - 1)(x^4 + x^3 + x^2 + x + 1)$

v. $(x-1)(x^n+x^{n-1}+\cdots+x^3+x^2+x+1)$

b. Substitute $x=10$ into each of the products from parts (i) through (iv) and your answers to show how each of the products appears as a statement in arithmetic.

c. If we substituted $x=10$ into the product $(x-2)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$ and computed the product, what number would result?

d. Multiply $(x-2)$ and $(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$, and express your answer in standard form.

Substitute $x=10$ into your answer, and see if you obtain the same result that you obtained in part (c).

e. Francois says $(x-9)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$ must equal $x^7+x^6+x^5+x^4+x^3+x^2+x+1$ because when $x=10$, multiplying by $x-9$ is the same as multiplying by 1.

i. Multiply $(x-9)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$.

ii. Put $x=10$ into your answer.

Is it the same as $x^7+x^6+x^5+x^4+x^3+x^2+x+1$ with $x=10$?

iii. Was Francois right?

A Progression Toward Mastery

Assessment Task Item		STEP 1 Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	STEP 2 Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	STEP 3 A correct answer with some evidence of reasoning or application of mathematics to solve the problem <u>or</u> an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	STEP 4 A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	**a** N-Q.A.1 N-Q.A.2	Student is unable to respond to question. OR Student provides a minimal attempt to create an incorrect graph.	Student creates a graph that reflects something related to the problem, but the axes do not depict the correct units of distance from the house on the y-axis and a measurement of time on the x-axis, or the graph indicates significant errors in calculations or reasoning.	Student creates axes that depict distance from the house on the y-axis and some measurement of time on the x-axis, and the graph represents a reflection of what occurred but with errors in calculations, missing or erroneous axis labels, or choice of units that makes the graph difficult to obtain information from.	Student creates and labels the y-axis to represent distance from the house in miles and an x-axis to represent time (in minutes past 1:00 p.m.) and creates a graph based on solid reasoning and correct calculations.

	b **N-Q.A.1**	Student answers incorrectly with no evidence of reasoning to support the answer. OR Student leaves item blank.	Student answers incorrectly but demonstrates some reasoning in explaining the answer.	Student answers 1:21 p.m. but does not either refer to a correct graph or provide sound reasoning to support the answer. OR Student answers incorrectly because either the graph in part (a) is incorrect, and the graph is referenced or because a minor calculation error is made, but sound reasoning is used.	Student answers 1:21 p.m. and either refers to a correct graph from part (a) or provides reasoning and calculations to explain the answer.
	c **N-Q.A.1**	Student answers incorrectly with no evidence of reasoning to support the answer. OR Student leaves item blank.	Student answers incorrectly but demonstrates some reasoning in explaining the answer.	Student answers 12 miles but does not either refer to the work in part (a) or provide sound reasoning in support of the answer. OR Student answers incorrectly because either the work in part (a) is referenced, but the work is incorrect or because a minor calculation error is made but sound reasoning is used.	Student answers 12 miles and either references correct work from part (a) or provides reasoning and calculations to support the answer.

2	**a** **N-Q.A.3**	Student leaves the question blank. OR Student provides an answer that reflects no or very little reasoning.	Student either begins with an assumption that is not based on the evidence of water being used at a rate of approximately 10 liters/second at noon. OR Student uses poor reasoning in extending that reading to consider total use across 24 hours.	Student answers beginning with the idea that water is being used at a rate of approximately 10 liters/second at noon but makes an error in the calculations to extend and combine that rate to consider usage across 24 hours. OR Student does not defend the choice by explaining water usage across the 24 hours and how it compares to the reading taken at noon.	Student answers beginning with the idea that water is being used at a rate of approximately 10 liters/second at noon and makes correct calculations to extend and combine that rate to consider usage across 24 hours. AND Student defends the choice by explaining water usage across the 24 hours and how it compares to the reading taken at noon.
	b **N-Q.A.3**	Student leaves the question blank. OR Student provides an answer that reflects no or very little reasoning.	Student provides an answer that is outside of the range from *to the nearest ten* to *to the nearest hundred*. OR Student provides an answer that is within the range but is not supported by an explanation.	Student answer ranges from *to the nearest ten* to *to the nearest hundred* but is not well supported by sound reasoning. OR Student answer contains an error in the way the explanation is written, even if it is clear what the student means to say.	Student answer ranges from *to the nearest ten* to *to the nearest hundred* and is supported by correct reasoning that is expressed accurately.

	c **N-Q.A.3**	Student leaves the question blank. OR Student provides an answer that reflects no or very little reasoning.	Student answer is not in the range of 6 to 48 checks but provides some reasoning to justify the choice. OR Student answer is in that range, perhaps written in the form of *every x minutes* or *every x hours* but is not supported by an explanation with solid reasoning.	Student answer is in the range of 6 to 48 checks but is only given in the form of x checks per minute or x checks per hour; the answer is well supported by a written explanation. OR Student answer is given in terms of number of checks but is not well supported by a written explanation.	Student answer is in the range of 6 to 48 checks, and student provides solid reasoning to support the answer.
3	**a** **A-SSE.A.1a** **A-SSE.A.1b**	Student either does not answer. OR Student answers incorrectly for all three expressions.	Student answers one or two of the three correctly but leaves the other one blank or makes a gross error in describing what it represents.	Student answers two of the three correctly and makes a reasonable attempt at describing what the other one represents.	Student answers all three correctly.
	b **A-SSE.A.1a** **A-SSE.A.1b**	Student either does not answer. OR Student answers incorrectly for all three parts of the question.	Student understands that the expressions represent a portion of the orders for each color but mis-assigns the colors and/or incorrectly determines which one is larger.	Student understands that the expressions represent a portion of the orders for each color and correctly determines which one is larger but has errors in the way the answer is worded or does not provide support for why $\frac{n}{m+n}$ is larger.	Student understands that the expressions represent a portion of the orders for each color, correctly determines which one is larger, and provides a well-written explanation for why.

4	**A-SSE.A.1b** **A-SSE.A.2**	Student leaves the question blank. OR Student is unable to rewrite the expression successfully, even by multiplying out the factors first.	Student gets to the correct rewritten expression of $8x + 24$ but does so by multiplying out the factors first. OR Student does not show the work needed to demonstrate how $8x + 24$ is determined.	Student attempts to use structure to rewrite the expression as described, showing the process, but student makes errors in the process.	Student correctly uses the process described to arrive at $8x + 24$ without multiplying out linear factors and demonstrates the steps for doing so.
5	**a–b** **A-SSE.A.2**	Student is unable to respond to many of the questions. OR Student leaves several items blank.	Student is only able to come up with one option for part (a) and, therefore, has only partial work for part (b). OR Student answers *Yes* for the question about equivalent expressions.	Student successfully answers part (a) and identifies that the expressions created in part (b) are not equivalent, but there are minor errors in the answering of the remaining questions.	Student answers all four parts correctly and completely.
6	**a** **A-SSE.A.2**	Student leaves at least three items blank. OR Student answers at least three items incorrectly.	Student answers one or two items incorrectly or leaves one or more items blank.	Student completes circling task correctly and provides a correct ordering of symbols in the box, but the answer does not use parentheses or multiplication dots.	Student completes all four item correctly, including exact placement of parentheses and symbols for the box: $x \cdot (3 \cdot x)$.
	b **A-SSE.A.2**	Student does not complete either proof successfully.	Student attempts both proofs but makes minor errors in both. OR Student only completes one proof, with or without errors.	Student attempts both proofs but makes an error in one of them.	Student completes both proofs correctly, and the two proofs are different from one another.

7	**a** **A-APR.A.1**	Student leaves the question blank or demonstrates no understanding of multiplication of polynomials.	Student makes more than one error in his multiplication but demonstrates some understanding of multiplication of polynomials.	Student makes a minor error in the multiplication.	Student multiplies correctly and expresses the resulting polynomial as a sum of monomials.
	b **A-APR.A.1**	Student leaves the question blank or does not demonstrate a level of thinking that is higher than what is given in the problem's description of Ahmed's thinking.	Student uses language that does not indicate an understanding of base x and/or the place value system. Student may use language such as *shifting* or *moving*.	Student makes only minor errors in the use of mathematically correct language to relate the new number to the old in terms of place value and/or the use of base x.	Student makes no errors in the use of mathematically correct language to relate the new number to the old in terms of place value and/or the use of base x.
8	**a–c** **A-APR.A.1**	Student shows limited or no understanding of polynomial multiplication and of evaluating a polynomial for the given value of x.	Student makes multiple errors but shows some understanding of polynomial multiplication. Student may not combine like terms to present the product as the sum of monomials.	Student makes one or two minor errors but demonstrates knowledge of polynomial multiplication and combining like terms to create the new polynomial. Student also shows understanding of evaluating a polynomial for the given value of x.	Student completes all products correctly, expressing each as a sum of monomials with like terms collected and evaluated correctly when x is 10.
	d **A-APR.A.1**	Student shows limited or no understanding of polynomial multiplication and of evaluating a polynomial for the given value of x.	Student makes multiple errors but shows some understanding of polynomial multiplication. Student may combine like terms to present the product as the sum of monomials. Student get an incorrect result when evaluating with $x = 10$.	Student makes one or two minor errors but demonstrates knowledge of polynomial multiplication and combining like terms to create the new polynomial. Student also shows understanding of evaluating a polynomial for the given value of x.	Student correctly multiplies the polynomials and expresses the product as a polynomial in standard form. Student correctly evaluates with a value of 10 and answers *Yes*.

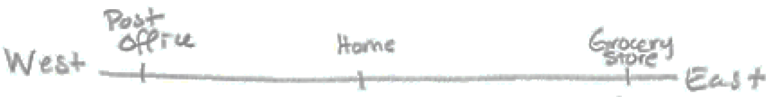
	e **A-APR.A.1**	Student is unable to demonstrate an understanding that part (iii) is *No* and/or demonstrates limited or no understanding of polynomial multiplication.	Student may make some errors as he multiplies the polynomials and expresses the product as a sum of monomials. Student may make some errors in the calculation of the value of the polynomial when x is 10. Student incorrectly answers part (iii) or applies incorrect reasoning.	Student may make minor errors in multiplying the polynomials and expressing the product as a sum of monomials. Student may make minor errors in calculating the value of the polynomial when x is 10. Student explains that the hypothesized equation being true when $x = 10$ does not make it true for all real x and/or explains that the two expressions are not algebraically equivalent.	Student correctly multiplies the polynomials and expresses the product as a sum of monomials with like terms collected. Student correctly calculates the value of the polynomial when x is 10. Student explains that the hypothesized equation being true when $x = 10$ does not make it true for all real x and/or explains that the two expressions are not algebraically equivalent.

Name ____ Date ____________________

1. Jacob lives on a street that runs east and west. The grocery store is to the east and the post office is to the west of his house. Both are on the same street as his house. Answer the questions below about the following story:

   At 1:00 p.m., Jacob hops in his car and drives at a constant speed of 25 mph for 6 minutes to the post office. After 10 minutes at the post office, he realizes he is late, and drives at a constant speed of 30 mph to the grocery store, arriving at 1:28 p.m. He then spends 20 minutes buying groceries.

   a. Draw a graph that shows the distance Jacob's car is from his house with respect to time. Remember to label your axes with the units you chose and any important points (home, post office, grocery store).



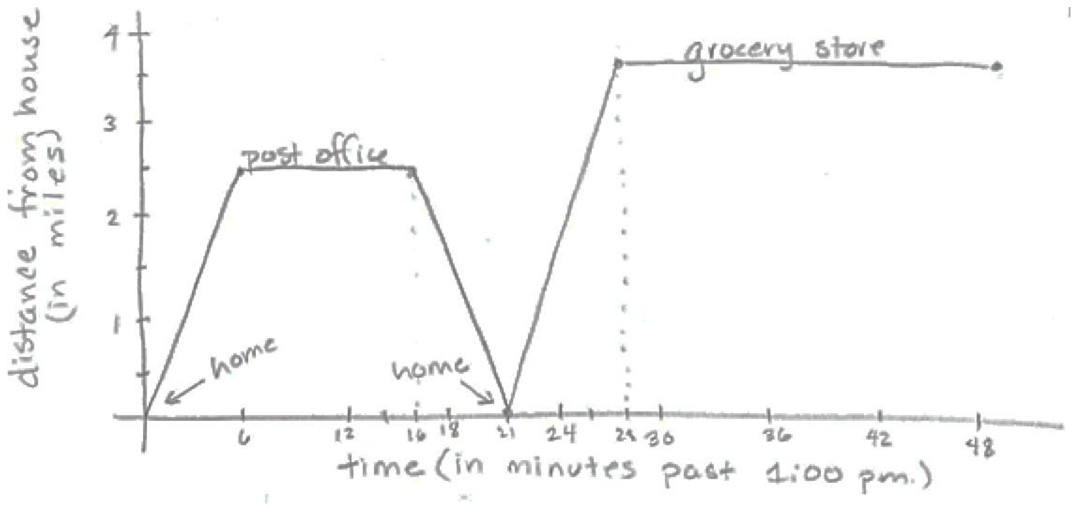
$$25\,\frac{\text{miles}}{\text{hour}} \times 6 \text{ minutes} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = 2.5 \text{ miles from house to post office}$$

$$30\,\frac{\text{miles}}{\text{hour}} \times 12 \text{ minutes} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = 6 \text{ miles from post office to store}$$

6 miles − 2.5 miles = 3.5 miles from home to store

6 miles in 12 minutes is 1 mile in 2 minutes

so 2. 5 miles takes 5 minutes and 3.5 miles takes 7 minutes

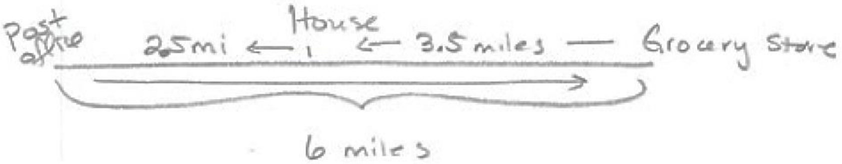


b. On the way to the grocery store, Jacob looks down at his watch and notes the time as he passes his house. What time is it when he passes his house? Explain how you found your answer.

It is 1:21 p.m. The graph shows the time as 21 minutes past 1:00 p.m. He spent 6 minutes getting to the post office, 10 minutes at the post office, and 5 minutes getting from the post office to the point of passing by his house. You know it took 5 minutes for the last part because he traveled 30 miles per hour and went 2.5 miles.

$2.5 \text{ miles} \times \frac{60 \text{ minutes}}{30 \text{ miles}} = 5 \text{ minutes}$

c. If he drives directly back to his house after the grocery story, what was the total distance he traveled to complete his errands? Show how you found your answer.



12 miles.

$2.5 \text{ miles} + 6 \text{ miles} + 3.5 \text{ miles} = 12 \text{ miles}$

You know it is 2.5 miles from the house to the post office because

$25 \frac{\text{miles}}{\text{hour}} \times 6 \text{ minutes} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = 2.5 \text{ miles.}$

You know it is 6 miles from the post office to the store because

$30 \frac{\text{miles}}{\text{hour}} \times 12 \text{ minutes} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = 6 \text{ miles.}$

2. Jason is collecting data on the rate of water usage in the tallest skyscraper in the world during a typical day. The skyscraper contains both apartments and businesses. The electronic water meter for the building displays the total amount of water used in liters. At noon, Jason looks at the water meter and notes that the digit in the ones place on the water meter display changes too rapidly to read the digit and that the digit in the tens place changes every second or so.

   a. Estimate the total number of liters used in the building during one 24-hour day. Take into account the time of day when he made his observation. (Hint: Will water be used at the same rate at 2:00 a.m. as at noon?) Explain how you arrived at your estimate.

   $$10\frac{\text{liters}}{\text{second}} \times 60\frac{\text{seconds}}{\text{minute}} \times 60\frac{\text{minutes}}{\text{hour}} \times 18 \text{ hours} = 648\,000 \text{ liters}$$

   Since water is probably only used from about 5:00 a.m. to 11:00 p.m., I did not multiply by 24 hours, but by 18 hours instead.

   b. To what level of accuracy can Jason reasonably report a measurement if he takes it at precisely 12:00 p.m.? Explain your answer.

   It can be reported within ±10 liters, since he can read the 10's place, but it is changing by a 10 during the second he reads it.

   c. The meter will be checked at regular time spans (for example, every minute, every 10 minutes, and every hour). What is the minimum (or smallest) number of checks needed in a 24-hour period to create a reasonably accurate graph of the water usage rate with respect to time? (For example, 24 checks would mean checking the meter every hour; 48 checks would mean checking the meter every half hour.) Defend your choice by describing how the water usage rate might change during the day and how your choice could capture that change.

   24 checks. Every hour would be good to show the peaks in usage during morning and evening hours from those in the apartments. And it might also show that businesses stop using it after business hours. It would depend on what portion of the building is business vs. apartments.

3. A publishing company orders black and blue ink in bulk for its two-color printing press. To keep things simple with its ink supplier, each time it places an order for blue ink, it buys B gallons, and each time it places an order for black ink, it buys K gallons. Over a one-month period, the company places m orders of blue ink and n orders of black ink.

   a. What quantities could the following expressions represent in terms of the problem context?

   $m + n$ Total number of ink orders over a one-month period.

   $mB + nK$ Total number of ink orders over a one-month period.

   $\frac{mB+nK}{m+n}$ Total number of ink orders over a one-month period.

   b. The company placed twice as many orders for black ink than for blue ink in January. Give interpretations for the following expressions in terms of the orders placed in January,

   $$\frac{m}{m+n} \quad \text{and} \quad \frac{n}{m+n},$$

   and explain which expression must be greater using those interpretations.

   $\frac{m}{m+n}$ is the fraction of orders that are for blue ink.

   $\frac{n}{m+n}$ is the fraction of orders that are for black ink.

   $\frac{n}{m+n}$ would be bigger, 2 times as big as $\frac{m}{m+n}$ because they ordered twice as many orders for black ink than for blue ink.

4. Sam says that he knows a clever set of steps to rewrite the expression

$$(x+3)(3x+8)-3x(x+3)$$

as a sum of two terms where the steps do not involve multiplying the linear factors first and then collecting like terms. Rewrite the expression as a sum of two terms (where one term is a number and the other is a product of a coefficient and variable) using Sam's steps if you can.

$\big((3x+8)-3x\big)\cdot(x+3)$

$8(x+3)$

$8x+24$

5. Using only the addition and multiplication operations with the numbers 1, 2, 3, and 4 each exactly once, it is possible to build a numeric expression (with parentheses to show the order used to build the expression) that evaluates to 21. For example, $1+\big((2+3)\cdot 4\big)$ is one such expression.

   a. Build two more numeric expressions that evaluate to 21 using the criteria above. Both must be different from the example given.

   $(1+2)\cdot(3+4)=21$

   $\big((2+4)+1\big)\cdot 3=21$

b. In both of your expressions, replace 1 with a, 2 with b, 3 with c, and 4 with d to get two algebraic expressions. For example, $a + \big((b + c) \cdot d\big)$ shows the replacements for the example given.

$$(a + b) \cdot (c + d) = ac + ad + bc + bd$$

$$\big((b + d) + a\big) \cdot c = ac + bc + dc$$

Are your algebraic expressions equivalent? Circle: Yes [✓ No]

- If they are equivalent, prove that they are using the properties of operations.
- If not, provide **two** examples:

  (1) Find four different numbers (other than 0, 1, 2, 3, 4) that when substituted for a, b, c, and d into each expression, the expressions evaluate to **different numbers**, and

  $a = 5 \quad b = 10 \quad c = 20 \quad d = 30$

  $$(5 + 10) \cdot (20 + 30) = 750$$

  $$\big((10 + 30) + 5\big) \cdot 20 = 900$$

  (2) Find four different, nonzero numbers that when substituted into each expression, the expressions evaluate to the **same number**.

  $5, 6, 11, 7$

  $(5 + 6) \cdot (11 + 7) = 11 \cdot 18 = 198$

  $\big((6 + 7) + 5\big) \cdot 11 = 18 \cdot 11 = 198$

  $(ac + ad + bc + bd)$ needs to equal $(ac + bc + dc)$;

  so, $(ad + bd)$ needs to equal (dc);

  so, $(a + b)$ needs to equal c.

6. The diagram below, when completed, shows all possible ways to build equivalent expressions of $3x^2$ using multiplication. The equivalent expressions are connected by labeled segments stating which property of operations, **A** for associative property and **C** for commutative property, justifies why the two expressions are equivalent. Answer the following questions about $3x^2$ and the diagram.

$$\begin{tikzcd}
 & (3\cdot x)\cdot x \arrow[r, leftrightarrow, "C"] \arrow[d, leftrightarrow, "C"'] & x\cdot(3\cdot x) \arrow[dl, leftrightarrow, "A"'] \arrow[d, leftrightarrow, "C"] \\
3\cdot(x\cdot x) \arrow[ur, leftrightarrow, "A"] \arrow[dr, leftrightarrow, "C"'] & (x\cdot 3)\cdot x \arrow[r, leftrightarrow, "C"'] & x\cdot(x\cdot 3) \\
 & (x\cdot x)\cdot 3 \arrow[ur, leftrightarrow, "A"'] &
\end{tikzcd}$$

a. Fill in the empty circles with **A** or **C** and the empty rectangle with the missing expression to complete the diagram.

b. Using the diagram above to help guide you, give *two different* proofs that $(x\cdot x)\cdot 3 = (3\cdot x)\cdot x$.

1. $(x\cdot x)\cdot 3 = x\cdot(x\cdot 3)$ by associative property

$x\cdot(x\cdot 3) = x\cdot(3\cdot x)$ by commutative property

$x\cdot(3\cdot x) = (3\cdot x)\cdot x$ by commutative property

2. $(x\cdot x)\cdot 3 = 3\cdot(x\cdot x)$ by commutative property

$3\cdot(x\cdot x) = (3\cdot x)\cdot x$ by associative property

7. Ahmed learned: “To multiply a whole number by ten, just place a zero at the end of the number.” For example, 2813×10, he says, is $28{,}130$. He doesn't understand why this *rule* is true.

   a. What is the product of the polynomial $2x^3 + 8x^2 + x + 3$ times the polynomial x?

   $2x^4 + 8x^3 + x^2 + 3x$

   b. Use part (a) as a hint. Explain why the rule Ahmed learned is true.

   When you multiply by the same number as the base, it creates a new number where each digit in the original number is now one place value higher so that there is nothing left (no numbers) to represent the ones digit, which leads to a trailing “0” in the ones digit.

8.

   a. Find the following products:

   i. $(x-1)(x+1)$

   $x^2 + x - x - 1$

   $x^2 - 1$

   ii. $(x-1)(x^2+x+1)$

   $x^3 + x^2 + x - x^2 - x - 1$

   $x^3 - 1$

   iii. $(x-1)(x^3+x^2+x+1)$

   $x^4 + x^3 + x^2 + x - x^3 - x^2 - x - 1$

   $x^4 - 1$

   iv. $(x-1)(x^4+\ x^3+x^2+x+1)$

   $x^5 + x^4 + x^3 + x^2 + x - x^4 - x^3 - x^2 - x - 1$

   $x^5 - 1$

v. $(x-1)(x^n + x^{n-1} + \cdots + x^3 + x^2 + x + 1)$

$x^{n+1} - 1$

b. Substitute $x = 10$ into each of the products from parts (i) through (iv) and your answers to show how each of the products appears as a statement in arithmetic.

i. $(10-1) \cdot (10+1) = (100-1)$

$9 \cdot (11) = 99$

ii. $(10-1) \cdot (100+10+1) = (1000-1)$

$9 \cdot (111) = 999$

iii. $(10-1) \cdot (1000+100+10+1) = (10000-1)$

$9 \cdot (1111) = 9{,}999$

iv. $(10-1) \cdot (10000+1000+100+10+1) = (100000-1)$

$9 \cdot (11111) = 99{,}999$

c. If we substituted $x = 10$ into the product $(x-2)(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$ and computed the product, what number would result?

$8 \cdot (11\,111\,111) = 88\,888\,888$

d. Multiply $(x-2)$ and $(x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$ and express your answer in standard form.

$x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x - 2x^7 - 2x^6 - 2x^5 - 2x^4 - 2x^3 - 2x^2 - 2x - 2$

$x^8 - x^7 - x^6 - x^5 - x^4 - x^3 - x^2 - x - 2$

Substitute $x = 10$ into your answer and see if you obtain the same result as you obtained in part (c).

$10^8 - 10^7 - 10^6 - 10^5 - 10^4 - 10^3 - 10^2 - 10 - 2 = 88\,888\,888$

Yes, I get the same answer.

e. Francois says $(x-9)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$ must equal $x^7+x^6+x^5+x^4+x^3+x^2+x+1$ because when $x=10$, multiplying by $x-9$ is the same as multiplying by 1.

i. Multiply $(x-9)(x^7+x^6+x^5+x^4+x^3+x^2+x+1)$.

$$x^8-8x^7-8x^6-8x^5-8x^4-8x^3-8x^2-8x-9$$

ii. Put $x=10$ into your answer.

$$100\,000\,000-80\,000\,000-8\,000\,000-800\,000-80000-8000-800-80-9$$

$$100\,000\,000-88\,888\,889=11\,111\,111$$

Is it the same as $x^7+x^6+x^5+x^4+x^3+x^2+x+1$ with $x=10$?

Yes.

iii. Was Francois right?

No, just because it is true when x is 10, doesn't make it true for all real x. The two expressions are not algebraically equivalent.