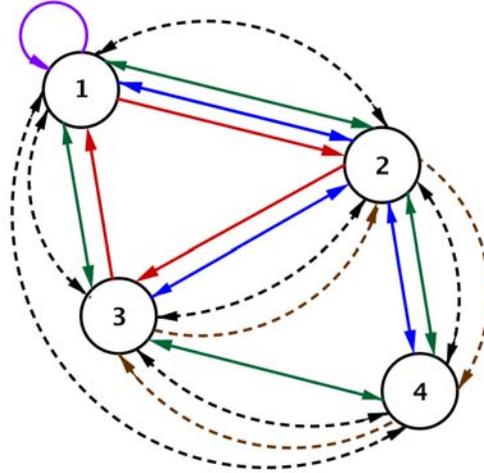**Lesson 2 - Opening Exercise**

Suppose a subway line also connects the four cities. Here is the subway and bus line network. The bus routes connecting the cities are represented by solid lines and the subway routes are represented by dashed arcs.

Write a matrix B to represent the bus routes and a matrix S to represent the subway lines connecting the four cities.

**Lesson 2 - Exploratory Challenge: Matrix Arithmetic**

Use the network diagram from the Opening Exercise and your answers to help you complete this challenge with your group.

1. Suppose the number of bus routes between each city were doubled.
    a. What would the new bus route matrix be?

    b. Mathematicians call this matrix $2B$. Why do you think they call it that?

2. What would be the meaning of $10B$ in this situation?

3. Write the matrix $10B$.

4. Ignore whether or not a line connecting cities represents a bus or subway route.

   a. Create one matrix that represents all the routes between the cities in this transportation network.

   b. Why would it be appropriate to call this matrix $B + S$? Explain your reasoning.

5. What would be the meaning of $4B + 2S$ in this situation?

6. Write the matrix $4B + 2S$. Show work and explain how you found your answer.

Lesson 3 - Opening Exercise

The subway and bus line network connecting four cities that we used in Lesson 2 is shown below. The bus routes connecting the cities are represented by solid lines and the subway routes are represented by dashed lines.
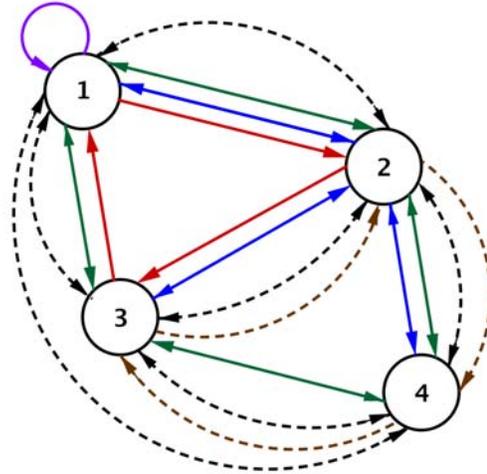
Suppose we want to travel from City 2 to City 1, first by bus and then by subway, with no more than one connecting stop.

1. Complete the chart below showing the number of ways to travel from City 2 to City 1 using first a bus and then the subway. The first row has been completed for you.

First Leg (BUS)		Second Leg (SUBWAY)		Total Ways to Travel
City 2 to City 1:	2	City 1 to City 1:	0	$2 \cdot 0 = 0$
City 2 to City 2:		City 2 to City 1:		
City 2 to City 3:		City 3 to City 1:		
City 2 to City 4:		City 4 to City 1:		

2. How many ways are there to travel from City 2 to City 1 first on a bus and then on a subway? How do you know?

Lesson 3 - Exploratory Challenge 2: The Meaning of Matrix Multiplication

Suppose we want to travel between the all cities, traveling first by bus and then by subway with no more than one connecting stop.

1. Use a chart like the one in the Opening Exercise 1 to help you determine the total number of ways to travel from City 1 to City 4 using first a bus and then the subway.

2. Suppose we create a new matrix P, to show the number of ways to travel between the cities first by bus and then by subway with no more than one connecting stop. Record your answers to Exercise 2 and Exercise 3 in this matrix in the appropriate row and column location. We do not yet have enough information to complete the entire matrix. Explain how you decided where to record these numbers in the matrix shown below.

$$P = \begin{bmatrix} \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \\ \_ & \_ & \_ & \_ \end{bmatrix}$$

3. What is the total number of ways to travel from City 3 to City 2 first by bus and then by subway with no more than one connecting stop? Explain how you got your answer and where you would record it in matrix P above.

Recall matrix B, which shows the number of bus lines connecting the cities in this transportation network, and matrix S, which represents the number of subway lines connecting the cities in this transportation network.

$$B = \begin{bmatrix} 1 & 3 & 1 & 0 \\ 2 & 0 & 2 & 2 \\ 2 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix} \text{ and } S = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 \\ 1 & 2 & 0 & 1 \\ 1 & 1 & 2 & 0 \end{bmatrix}$$

4. What does the product $b_{1,2}s_{2,4}$ represent in this situation? What is the value of this product?

5. What does $b_{1,4}s_{4,4}$ represent in this situation? What is the value of this product? Does this make sense?

6. Calculate the value of the expression $b_{1,1}s_{1,4} + b_{1,2}s_{2,4} + b_{1,3}s_{3,4} + b_{1,4}s_{4,4}$. What is the meaning of this expression in this situation?

7. Circle the first row of B and the second column of S. How are these entries related to the expression above and your work in Exercise 3?

8. Write an expression that represents the total number of ways you can travel between City 2 and City 1 first by bus and then by subway with no more than one connecting stop. What is the value of this expression? What is the meaning of the result?

9. Write an expression that represents the total number ways you can travel between City 4 and City 1 first by bus and then by subway with no more than one connecting stop. What is the value of this expression? What is the meaning of the result?

10. Complete matrix P that represents the routes connecting the four cities if you travel first by bus and then by subway.

11. Construct a matrix M that represents the routes connecting the four cities if you travel first by subway and then by bus.

12. Should these two matrices be the same? Explain your reasoning.

Lesson 7 -Exploratory Challenge 1

1. For each matrix A given below:

      i. Plot the image of the unit cube under the transformation.

      ii. Find the volume of the image of the unit cube from part (ii).

      iii. Does the transformation have an inverse? If so, what is the matrix that induces the inverse transformation?

    a. $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

    b. $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

    c. $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

    d. Describe the geometric effect of a transformation $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ for numbers a, b, and c. Describe when such a transformation is invertible.

Lesson 7 - Exploratory Challenge 2

2. **Make a prediction: What would be the geometric effect of the transformation**

$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(45°) & -\sin(45°) \\ 0 & \sin(45°) & \cos(45°) \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$ **on the unit cube? Use the GeoGebra demo to test your conjecture.**

3. **For each geometric transformation below, find a matrix A so that the geometric effect of $L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = A \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is the specified transformation.**

   a.   **Rotation by $-45°$ about the x-axis.**

   b.   **Rotation by $45°$ about the y-axis.**

   c.   **Rotation by $45°$ about the z-axis.**

   d.   **Rotation by $90°$ about the x-axis.**

   e.   **Rotation by $90°$ about the y-axis.**

   f.   **Rotation by $90°$ about the z-axis.**

   g.   **Rotation by θ about the x-axis.**

   h.   **Rotation by θ about the y-axis.**

   i.   **Rotation by θ about the z-axis.**

Lesson 9 - Opening Exercise

1. Recall from problem 1(d) of the Problem Set of Lesson 7 that if you know what a linear transformation does to the three points $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$, you can find the matrix of the transformation. How do the images of these three points lead to the matrix of the transformation?

   a. Suppose that a linear transformation L_1 rotates the unit cube by $90°$ counterclockwise about the z-axis. Find the matrix A_1 of the transformation L_1.

   b. Suppose that a linear transformation L_2 rotates the unit cube by $90°$ counterclockwise about the y-axis. Find the matrix A_2 of the transformation L_2.

   c. Suppose that a linear transformation L_3 scales by 2 in the x-direction, scales by 3 in the y-direction, and scales by 4 in the z-direction. Find the matrix A_3 of the transformation L_3.

   d. Suppose that a linear transformation L_4 projects onto the xy-plane. Find the matrix A_4 of the transformation L_4.

   e. Suppose that a linear transformation L_5 projects onto the xz-plane. Find the matrix A_5 of the transformation L_5.

   f. Suppose that a linear transformation L_6 reflects across the plane with equation $y = x$. Find the matrix A_5 of the transformation L_6.

g. Suppose that a linear transformation L_7 satisfies $L_7\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right)=\begin{bmatrix}2\\0\\0\end{bmatrix}$, $L_7\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right)=$

$\begin{bmatrix}0\\1\\0\end{bmatrix}$, and $L_7\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right)=\begin{bmatrix}0\\0\\\frac{1}{2}\end{bmatrix}$. Find the matrix A_7 of the transformation L_7. What is

the geometric effect of this transformation?

h. Suppose that a linear transformation L_8 satisfies $L_8\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right)=\begin{bmatrix}1\\1\\0\end{bmatrix}$, $L_8\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right)=$

$\begin{bmatrix}-1\\1\\0\end{bmatrix}$, and $L_8\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right)=\begin{bmatrix}0\\0\\1\end{bmatrix}$. Find the matrix of the transformation L_8. What is the

geometric effect of this transformation?

Lesson 9 - Exploratory Challenge 1

Opening Exercise

1. Recall from problem 1(d) of the Problem Set of Lesson 7 that if you know what a linear transformation does to the three points $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$, you can find the matrix of the transformation. How do the images of these three points lead to the matrix of the transformation?

    a. Suppose that a linear transformation L_1 rotates the unit cube by $90°$ counterclockwise about the z-axis. Find the matrix A_1 of the transformation L_1.

    b. Suppose that a linear transformation L_2 rotates the unit cube by $90°$ counterclockwise about the y-axis. Find the matrix A_2 of the transformation L_2.

c. Suppose that a linear transformation L_3 scales by 2 in the x-direction, scales by 3 in the y-direction, and scales by 4 in the z-direction. Find the matrix A_3 of the transformation L_3.

d. Suppose that a linear transformation L_4 projects onto the xy-plane. Find the matrix A_4 of the transformation L_4.

e. Suppose that a linear transformation L_5 projects onto the xz-plane. Find the matrix A_5 of the transformation L_5.

f. Suppose that a linear transformation L_6 reflects across the plane with equation $y = x$. Find the matrix A_5 of the transformation L_6.

g. Suppose that a linear transformation L_7 satisfies $L_7\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}2\\0\\0\end{bmatrix}, L_7\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}0\\1\\0\end{bmatrix}$, and $L_7\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}0\\0\\\frac{1}{2}\end{bmatrix}$. Find the matrix A_7 of the transformation L_7. What is the geometric effect of this transformation?

h. Suppose that a linear transformation L_8 satisfies $L_8\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}1\\1\\0\end{bmatrix}, L_8\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}-1\\1\\0\end{bmatrix}$, and $L_8\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}0\\0\\1\end{bmatrix}$. Find the matrix of the transformation L_8. What is the geometric effect of this transformation?

Transformations L_1-L_8 refer to the linear transformations from the Opening Exercise. For each pair,

    i.   Make a conjecture to predict the geometric effect of performing the two transformations in the order specified. – MP.3

    ii.   Find the product of the corresponding matrices, in the order that corresponds to the indicated order of composition. Remember that if we perform a transformation L_B with matrix B and then L_A with matrix A, the matrix that corresponds to the composition $L_A \circ L_B$ is AB. That is, L_B is applied first, but matrix B is written second.

    iii.   Use the GeoGebra applet TransformCubes.ggb to draw the image of the unit cube under the transformation induced by the matrix product in part (ii). Was your conjecture in part (i) correct?

a.   Perform L_6 and then L_6.

b.   Perform L_1 and then L_2..

c.   Perform L_4 and then L_5.

d.   Perform L_4 and then L_3.

e. Perform L_3 and then L_7.

f. Perform L_8 and then L_4.

g. Perform L_4 and then L_6.

h. Perform L_2 and then L_7.

i. Perform L_8 and then L_8.

Lesson 10 - Exercise 4

Write two matrices in the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$.

a. Verify algebraically that the product of these two matrices is equal.

b. Write each of your matrices as a complex number. Find the product of the two complex numbers.

c. Why is it the case that any two matrices in the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ have products that are equal regardless of the order in which they are multiplied?

Lesson 12 – Exercise 1

4. Let $A = \begin{bmatrix} x & z \\ y & w \end{bmatrix}$, B $= \begin{bmatrix} a & c \\ b & d \end{bmatrix}$, and $C = \begin{bmatrix} e & g \\ f & h \end{bmatrix}$.

    a. Write down the products $AB, AC, and\ A(B + C)$.

    b. Verify that $A(B + C) = AB + AC$.

Lesson 13 - Example (10 minutes)

- A common way to send coded messages is to assign each letter of the alphabet to a number 1-26 and send the message as a string of integers. For example, if we encode the message "THE CROW FLIES AT MIDNIGHT" according to the chart below, we get the string of numbers

$$20, 8, 5, 0, 3, 18, 15, 23, 0, 6, 12, 9, 5, 19, 0, 1, 20, 0, 13, 9, 4, 14, 9, 7, 8, 20.$$

A	B	C	D	E	F	G	H	I	J	K	L	M
1	2	3	4	5	6	7	8	9	10	11	12	13

N	O	P	Q	R	S	T	U	V	W	X	Y	Z	SPACE
14	15	16	17	18	19	20	21	22	23	24	25	26	0

However, codes such as these are easily broken using an analysis of the frequency of numbers that appear in the coded messages. For example if the coded phrase is $20, 8, 5, 0, 3, 18, 15, 23, 0, 6, 12, 9, 5, 19, 0, 1, 20, 0, 13, 9, 4, 14, 9, 7, 8, 20$, we can see that there are three of the letter assigned to the integer 20, so we may want to try putting a common letter in for the number 20 like S, T, or E. If we assume the word "the" is used to start the phrase since we have 3 letters then a space, that would lead us to maybe the number 5 is E, and so on.

- We can instead encode a message using matrix multiplication. If a matrix E has an inverse, then we can encode a message as follows.

- First, convert the characters of the message to integers between 1 and 26 using the chart above.

- If the encoding matrix E is an $n \times n$ matrix, then break up the numerical message into n rows of the same length. If needed, add extra zeros to make the rows the same length. For example if we want to use an encoding matrix that is 2×2, we would write the message in two rows of equal length, filling in zero for the last number if the number of letters was odd. If the encoding matrix is 3×3, the message would be written in 3 equal rows adding zeros as necessary.

- Place the rows into a matrix M.

- Compute the product EM to encode the message.

- The message is sent as the numbers in the rows of the matrix EM.

- Using the matrix $E = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$, we encode our message as follows:

- We've already converted our message "THE CROW FLIES AT MIDNIGHT" to a string of numbers:

$$20, 8, 5, 0, 3, 18, 15, 23, 0, 6, 12, 9, 5, 19, 0, 1, 20, 0, 13, 9, 4, 14, 9, 7, 8, 20.$$

- Since E is a 2×2 matrix, we need to break up our message into two rows.

$$20, 8, 5, 0, 3, 18, 15, 23, 0, 6, 12, 9, 5,$$
$$19, 0, 1, 20, 0, 13, 9, 4, 14, 9, 7, 8, 20.$$

- Then we place the rows into a matrix M.

$$M = \begin{bmatrix} 20 & 8 & 5 & 0 & 3 & 18 & 15 & 23 & 0 & 6 & 12 & 9 & 5 \\ 19 & 0 & 1 & 20 & 0 & 13 & 9 & 4 & 14 & 9 & 7 & 8 & 20 \end{bmatrix}.$$

- Explain how matrix M represent "the crow flies at midnight". – MP.2

  o *Each letter and space in the phrase was assigned an integer value and these numbers represent the letters in the phrase.*

- We encode the message into matrix C by multiplying $E \cdot M$.

$$C = E \cdot M = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 20 & 8 & 5 & 0 & 3 & 18 & 15 & 23 & 0 & 6 & 12 & 9 & 5 \\ 19 & 0 & 1 & 20 & 0 & 13 & 9 & 4 & 14 & 9 & 7 & 8 & 20 \end{bmatrix}$$
$$C = \begin{bmatrix} 59 & 16 & 11 & 20 & 6 & 49 & 39 & 50 & 14 & 21 & 31 & 26 & 30 \\ 79 & 24 & 16 & 20 & 9 & 67 & 54 & 73 & 14 & 27 & 43 & 35 & 35 \end{bmatrix}.$$

- Thus, the coded message that we send is

  $59, 16, 11, 20, 6, 49, 39, 50, 14, 21, 31, 26, 30, 79, 24, 16, 20, 9, 67, 54, 73, 14, 27, 43, 35, 35.$

  If this coded message is intercepted, then it cannot easily be decoded unless the recipient knows how it was originally encoded.

Be sure to work through this discussion and emphasize that the way to decode a message is to multiply by the inverse of the encoding message.

- Using what you know about how the message was encoded, as well as matrix multiplication, describe how you would decode this message. – MP.4

  □ *We need to know a decoding matrix D.*

- How can we find that matrix?

  □ *The decoding matrix is the inverse of the encoding matrix, so $D = E^{-1}$.*

- What is the decoding matrix?

  □ $D = E^{-1} = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}^{-1} = \frac{1}{2-3} \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}.$

- Decode this message!

  □ $D \cdot C = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 59 & 16 & 11 & 20 & 6 & 49 & 39 & 50 & 14 & 21 & 31 & 26 & 30 \\ 79 & 24 & 16 & 20 & 9 & 67 & 54 & 73 & 14 & 27 & 43 & 35 & 35 \end{bmatrix}$
  $= \begin{bmatrix} 20 & 8 & 5 & 0 & 3 & 18 & 15 & 23 & 0 & 6 & 12 & 9 & 5 \\ 19 & 0 & 1 & 20 & 0 & 13 & 9 & 4 & 14 & 9 & 7 & 8 & 20 \end{bmatrix}.$

  As expected, this is the matrix M that stored our original message "THE CROW FLIES AT MIDNIGHT."

- Why does this process work?

  □ *The coded message stored in matrix C is the product of matrices E and M, so $C = E \cdot M$. We then decode the message stored in matrix C by multiplying by matrix D. Since matrices D and E are inverses, we have*

  $$D \cdot (E \cdot M) = (D \cdot E) \cdot M = I \cdot M = M,$$

  so encoding and then decoding will return the original message in matrix M.

▪ **Explain to your neighbor what you learned about how to encode and decode messages. Teachers should use this as an informal way to check for understanding.**

Lesson 13 - Exercise 1

1. **You have received an encoded message:** $34, 101, 13, 16, 23, 45, 10, 8, 15, 50, 8, 12$.
   You know that the message was encoded using matrix $E = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

   a. **Store your message in a matrix** C. **What are the dimensions of** C?

   b. **You have forgotten whether the proper decoding matrix is matrix** $X, Y,$ **or** Z **as shown below. Determine which of these is the correct matrix to use to decode this message.**

   $$X = \begin{bmatrix} -1/3 & -1/3 & 4/3 \\ -1/3 & 2/3 & 1/3 \\ -2/3 & 1/3 & -2/3 \end{bmatrix}, Y = \begin{bmatrix} -1/3 & -1/3 & 4/3 \\ -1/3 & 2/3 & -1/3 \\ 2/3 & -1/3 & 2/3 \end{bmatrix},$$

   $$Z = \begin{bmatrix} -1/3 & -1/3 & 4/3 \\ -1/3 & 2/3 & 1/3 \\ 2/3 & -1/3 & -2/3 \end{bmatrix}$$

   c. **Decode the message.**

Lesson 15 – Exercise 2

2. **An Athletic Director at an all-boys high school is trying to find out how many coaches to hire for the football, basketball, and soccer teams. To do this, he needs to know the number of boys that play each sport. He does not have names or numbers, but find a note with the following information listed:**

   The total number of boys on all three teams is 76.

   The number of boys that play football is 7 less than double the total number of boys playing the other two sports.

   The number of boys that play football is 5 times the number of boys playing basketball.

   a. **Write a system of equations representing the number of boys playing each sport where x is the number of boys playing football, y basketball, and z soccer.**

   b. **Solve the system using algebra or matrix operations. If you use matrix operations, include the matrices you entered into the software and the calculations you performed to solve the system.**

   c. **Verify your solution is correct.**

   d. **Justify your decision to use the method you selected to solve the system.**

Lesson 16 - Exercise 1

1. An attendance officer in a small school district noticed a trend among the four elementary schools in the district. This district used an open enrollment policy, which means any student within the district could enroll at any school in the district. Each year, 10% of the students from Adams Elementary enrolled at Davis Elementary, and 10% of the students from Davis enrolled at Adams. In addition, 10% of the students from Brown Elementary enrolled at Carson Elementary, and 20% of the students from Brown enrolled at Davis. At Carson Elementary, about 10% of students enrolled at Brown and 10% enrolled at Davis, while at Davis, 10% enrolled at Brown and 20% enrolled at Carson. The officer noted that this year, the enrollment was $490, 250, 300$, and 370 at Adams, Brown, Carson, and Davis, respectively.

   a. Represent the relationship that reflects the annual movement of students among the elementary schools using a matrix.

   b. Write an expression that could be used to calculate the attendance one year prior to the year cited by the attendance officer. Find the enrollment for that year.

   c. Assuming that the trend in attendance continues, write an expression that could be used to calculate the enrollment two years after the year cited by the attendance officer. Find the attendance for that year.

   d. Interpret the results to part (c) in context.

Lesson 20 – Exploratory Challenge

1. **For this Exploratory Challenge, we will consider an arch made with 5 trapezoidal stones as shown.**
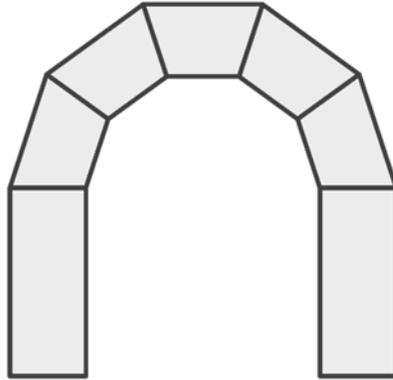
    a. **Find the magnitude and direction of each of the force vectors acting on the five stones of the arch. Draw these vectors on the arch above.**

    b. **Use the tip-to-tail addition method to find the sum of the force vectors on the left side of the arch, and the sum of the force vectors on the right side of the arch. Does it look like the arch will stand?**

    c. **Find coordinate representations of each of the force vectors in your diagram.**

    d. **Add the vectors component-wise to find the sum of the force vectors on the left side of the arch, and the sum of the force vectors on the right side of the arch. How much can the stones weigh so that an arch with a base column of height xxx will be stable?**

    e. **Add buttresses to double the width of the base column using your answer from part (d) for the weight of the stones, how tall can the base columns be for the arch to remain stable?**

Lesson 21 – Exercises 1 - 6

1. Consider the line ℓ in the plane given by the equation $3x - 2y = 6$.

    a. Sketch a graph of line ℓ on the axes provided.
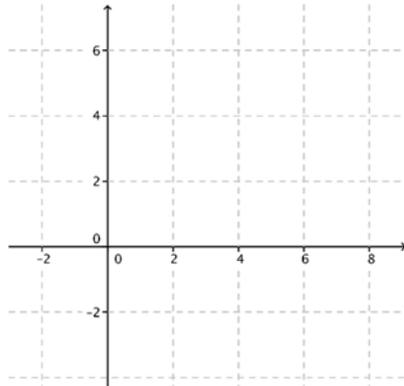
    b. Find a point on line ℓ and the slope of line ℓ.

    c. Write a vector equation for line ℓ using the information you found in Part (b).

    d. Write parametric equations for line ℓ.

    e. Verify algebraically that your parametric equations produce points on line ℓ.

2. Olivia wrote parametric equations $x(t) = 4 + 2t$ and $y(t) = 3 + 3t$. Are her equations correct? What did she do differently from you?

3. **Convert the parametric equations $x(t) = 2 - 3t$, $y(t) = 4 + t$ into slope-intercept form.**

4. **Find parametric equations to represent the line that passes through point $(4, 2, 9)$ and has direction vector $\vec{v} = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$.**

5. **Find a vector form of the equation of the line given by the parametric equations**
$$x(t) = 3t$$
$$y(t) = -4 - 2t$$
$$z(t) = 3 - t.$$

Lesson 22 - Exercise 1

1. **Consider points $P(2, 1, 4)$ and $Q(3, -1, 2)$, and define a linear transformation by**
$$L\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ 3 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$ **Find parametric equations to describe the image of line $\overleftrightarrow{PQ}$ under the transformation L.**

Lesson 23 - Example 1

An airplane flying from Dallas-Fort Worth to Atlanta veers off course to avoid a storm. The plane leaves Dallas-Fort Worth traveling $50°$ east of north and flies for 450 miles before turning to travel $70°$ east of south for 350 miles. What is the resultant displacement of the airplane? Include both the magnitude and direction of the displacement.

Lesson 23 - Exercise 1

1. A motorized robot moves across the coordinate plane. Its position $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ at time t seconds is given by $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = a + tv$ where $a = \begin{pmatrix} 4 \\ -10 \end{pmatrix}$ and $v = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$. The units of distance are measured in meters.

    a. Where is the robot at time $t = 0$?

    b. Plot the path of the robot.
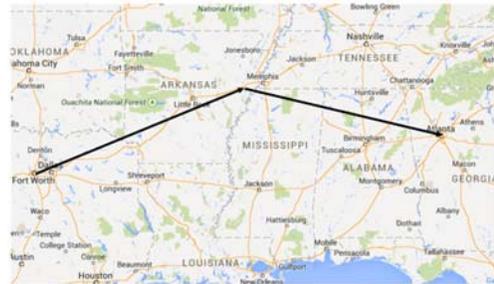
    c. Describe the path of the robot.

    d. Where is the robot 10 seconds after it starts moving?

    e. Where is the robot when it is 10 meters from where it started?

      f.   **Is the robot traveling at a constant speed? Explain, and if the speed is constant, state the robot's speed.**

Lesson 25 - Exercise 1

In this drawing task, the "eye" or the "camera" is the point, and the shaded figure is the "TV Screen." The cube is in the 3-D universe of the computer game.

By using lines drawn from each vertex of the cube to the point, draw the image of the 3-D cube on the screen. A horizon line and two additional vanishing points have been included to help you. The image point of the first vertex is shown.
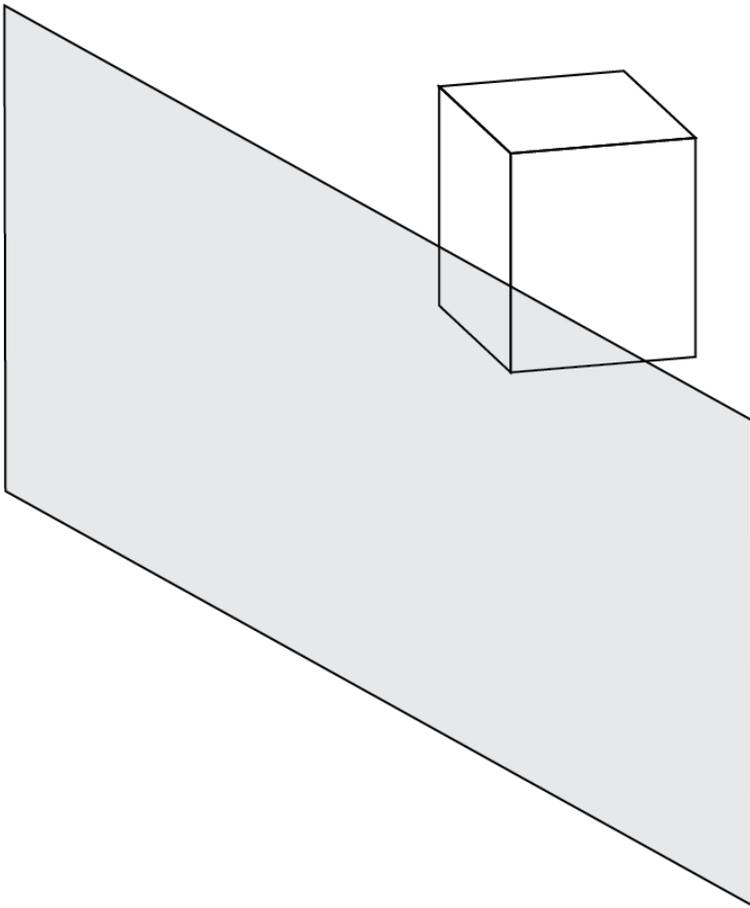
Lesson 26 - Example 1

a. Select the Sea Surface as your background scene, and select the setup scene mode. From the Gallery By Class Hierarchy, select Swimmer Classes, then Marine Mammals, and a Dolphin. Name your dolphin. Describe what you see.

b. Click on the dolphin, and from the handle style buttons in the top right corner of the screen, select translation. By selecting the arrows on the dolphin, we can move it to different locations in the screen. Move the dolphin left and right, up and down, forward and backward. Then, move it so that its coordinates are $(2, 0.2, 1)$.

c. Click on the rotation button from the handle style buttons, and practice rotating the dolphin about the three axes through its pivot point. Then rotate the dolphin so it is upright (with its back to the sky) and its nose faces right.

d. Select Edit Code from the screen. Drag and drop this.dolphinturn from the procedures menu and drop it into the declare procedures region on the right. Select from the drop-down menus: this.dolphin turn LEFT 1.0 as seen by this duration 2.0 BEGIN_AND_END_ABRUPTLY. Then run the program. Describe what you see.

e. Drag and drop this.dolphinroll from the procedures menu and drop it into the declare procedures region on the right beneath the turn procedure. Select from the drop-down menus: this.dolphin roll RIGHT 1.0 as seen by this duration 2.0 BEGIN_AND_END_ABRUPTLY. Then run the program. Describe what you see.