

Topic B

Decimal Expansions of Numbers

8.NS.A.1, 8.NS.A.2, 8.EE.A.2

Focus Standards:	8.NS.A.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
	8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). <i>For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue to get better approximations.</i>
	8.EE.A.2	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
Instructional Days:	9	
	Lesson 6:	Finite and Infinite Decimals (P) ¹
	Lesson 7:	Infinite Decimals (S)
	Lesson 8:	The Long Division Algorithm (E)
	Lesson 9:	Decimal Expansions of Fractions, Part 1 (P)
	Lesson 10:	Converting Repeating Decimals to Fractions (P)
	Lesson 11:	The Decimal Expansion of Some Irrational Numbers (S)
	Lesson 12:	Decimal Expansions of Fractions, Part 2 (S)
	Lesson 13:	Comparing Irrational Numbers (E)
	Lesson 14:	Decimal Expansion of π (S)

¹Lesson Structure Key: **P**-Problem Set Lesson, **M**-Modeling Cycle Lesson, **E**-Exploration Lesson, **S**-Socratic Lesson

Throughout this topic, the terms *expanded form of a decimal* and *decimal expansion* are used. The expanded form of a decimal refers to the value of a number written as a sum. For example, the expanded form of the decimal 0.125 is $\frac{1}{10} + \frac{2}{10^2} + \frac{5}{10^3}$, which is closely related to the notion of expanded form used at the elementary level. When students are asked to determine the decimal expansion of a number such as $\sqrt{2}$, we expect them to write the number in decimal form. For example, the decimal expansion of $\sqrt{2}$ begins with 1.4142. The examination of the decimal expansion leads to an understanding of irrational numbers. Numbers with decimal expansions that are infinite (i.e., non-terminating) and that do not have a repeating block are called *irrational numbers*. Numbers with finite (i.e., terminating) decimal expansions, as well as those numbers that are infinite with repeating blocks, are called *rational numbers*. Students spend significant time engaging with finite and infinite decimals before the notion of an irrational number is fully explored in Lesson 11.

In Lesson 6, students learn that every number has a decimal expansion that is finite or infinite. Finite and infinite decimals are defined, and students learn a strategy for writing a fraction as a finite decimal that focuses on the denominator and its factors. That is, a fraction can be written as a finite decimal if the denominator is a product of twos, a product of fives, or a product of twos and fives. In Lesson 7, students learn that numbers that cannot be expressed as finite decimals are infinite decimals. Students write the expanded form of infinite decimals and show on the number line their decimal representation in terms of intervals of tenths, hundredths, thousandths, and so on. This work with infinite decimals prepares students for understanding how to approximate the decimal expansion of an irrational number. In Lesson 8, students use the long division algorithm to determine the decimal form of a number and can relate the work of the algorithm to why digits in a decimal expansion repeat. Students engage in a discussion about numbers that have an infinite decimal expansion with no discernable pattern in the digits, leading to the idea that numbers can be irrational. It is in these first few lessons of Topic B that students recognize that rational numbers have a decimal expansion that repeats eventually, either in zeros or in a repeating block of digits. The discussion of infinite decimals continues with Lesson 9, where students learn how to use what they know about powers of 10 and equivalent fractions to make sense of why the long division algorithm can be used to convert a fraction to a decimal. Students know that multiplying the numerator and denominator of a fraction by a power of 10 is similar to putting zeros after the decimal point when doing long division.

In Lesson 10, students learn that a number with a decimal expansion that repeats can be expressed as a fraction. Students learn a strategy for writing repeating decimals as fractions that relies on their knowledge of multiplying by powers of 10 and solving linear equations. Lesson 11 introduces students to the method of rational approximation using a series of rational numbers to get closer and closer to a given number. Students write the approximate decimal expansion of irrational numbers in Lesson 11, and it is in this lesson that irrational numbers are defined as numbers that are not equal to rational numbers. Students realize that irrational numbers are different because they have infinite decimal expansions that do not repeat. Therefore, irrational numbers are those that are not equal to rational numbers. Rational approximation is used again in Lesson 12 to verify the decimal expansions of rational numbers. Students then compare the method of rational approximation to long division. In Lesson 13, students compare the value of rational and irrational numbers. Students use the method of rational approximation to determine the decimal expansion of an irrational number. Then, they compare that value to the decimal expansion of rational numbers in the form of a fraction, decimal, perfect square, or perfect cube. Students can now place irrational numbers on a number line with more accuracy than they did in Lesson 2. In Lesson 14, students approximate π using the area of a quarter circle that is drawn on grid paper. Students estimate the area of the quarter circle using inner and outer boundaries. As with the method of rational approximation, students continue to refine their

estimates of the area, which improves their estimate of the value of π . Students then determine the approximate values of expressions involving π .