



## Lesson 23: Newton's Law of Cooling

### Student Outcomes

- Students apply knowledge of exponential functions and transformations of functions to a contextual situation.

### Lesson Notes

Newton's law of cooling is a complex topic that appears in physics and calculus. Here, it is simplified to focus on the idea of applying the transformations learned in Lessons 17–20 to a contextual situation. The mathematical practice in focus for this lesson is again MP.4, modeling with mathematics.

### Classwork

#### Opening Exercise (7 minutes)

The Opening Exercise introduces students to the formula used in the lesson. Allow students a few minutes to read through the scenario and examine the given formula. Before they begin working, discuss each parameter of the formula as a class.

- What does  $T_a$  represent?  $T_0$ ?  $k$ ?  $T(t)$ ?

Give students time to complete the problem, and then discuss results.

- Why would it be important to be able to find the precise time of death?
  - To establish suspects, verify alibis, determine cause of death, etc.*

#### Opening Exercise

A detective is called to the scene of a crime where a dead body has just been found. He arrives at the scene and measures the temperature of the dead body at 9:30 p.m. After investigating the scene, he declares that the person died 10 hours prior, at approximately 11:30 a.m. A crime scene investigator arrives a little later and declares that the detective is wrong. She says that the person died at approximately 6:00 a.m., 15.5 hours prior to the measurement of the body temperature. She claims she can prove it by using Newton's law of cooling:

$$T(t) = T_a + (T_0 - T_a) \cdot 2 \cdot 718^{-kt},$$

where:

$T(t)$  is the temperature of the object after a time of  $t$  hours has elapsed,

$T_a$  is the ambient temperature (the temperature of the surroundings), assumed to be constant, not impacted by the cooling process,

$T_0$  is the initial temperature of the object, and

$k$  is the decay constant.

#### Scaffolding:

If students have difficulty inputting the formula in their calculators, encourage them to input the expression in separate steps, following the order of operations.

Using the data collected at the scene, decide who is correct: the detective or the crime scene investigator.

$$T_a = 68^\circ\text{F} \text{ (the temperature of the room)}$$

$$T_0 = 98.6^\circ\text{F} \text{ (the initial temperature of the body)}$$

$$k = 0.1335 \text{ (13.35\% per hour—calculated by the investigator from the data collected)}$$

The temperature of the body at 9:30 p.m. is  $72^\circ\text{F}$ .

*If the person had died 10 hours prior to 9:30 p.m., his body temperature would be approximately  $76^\circ\text{F}$ . When you fill in 15.5 hours for  $t$ , the temperature is  $71.865^\circ\text{F}$ . This is much closer to the measured temperature of  $72^\circ\text{F}$ .*

### Discussion (3 minutes)

Display the demonstration on Newton's law of cooling found at Wolfram Alpha:

<http://demonstrations.wolfram.com/NewtonsLawOfCooling/>

(Note that Wolfram's free CDF player needs to be downloaded ahead of time in order to be able to run the demonstration.)

- First, at what type of graph are we looking?
  - *An exponential decay curve that has been vertically translated by an initial value*
- Why is it still an exponential decay function when the base is greater than 1?
  - *Because the base is being raised to a negative power (graph is reflected around the y-axis)*

Allow students to drag the sliders and observe the effect of each on the graph. Keep this discussion fairly general. Students explore this more in the modeling exercise in Module 5.

### Mathematical Modeling Exercise (25 minutes)

Allow students to work through the exercise in groups. Encourage students to use the language from previous lessons on transformations such as *translate*, *stretch*, and *shrink*. If students ask how  $k$  is determined, explain that finding  $k$  requires algebra that they learn in Algebra II (logarithms). So, for this exercise,  $k$  is given.

#### Mathematical Modeling Exercise

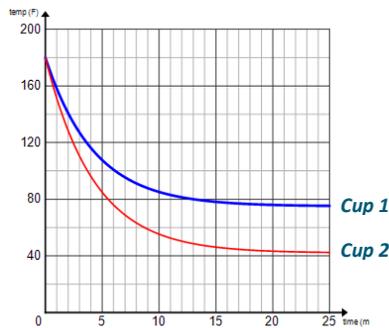
Two cups of coffee are poured from the same pot. The initial temperature of the coffee is  $180^\circ\text{F}$ , and  $k$  is 0.2337 (for time in minutes).

1. Suppose both cups are poured at the same time. Cup 1 is left sitting in the room that is  $75^\circ\text{F}$ , and Cup 2 is taken outside where it is  $42^\circ\text{F}$ .
  - a. Use Newton's law of cooling to write equations for the temperature of each cup of coffee after  $t$  minutes has elapsed.

$$\text{Cup 1: } T_1(t) = 75 + (180 - 75) \cdot 2.718^{-0.2337t}$$

$$\text{Cup 2: } T_2(t) = 42 + (180 - 42) \cdot 2.718^{-0.2337t}$$

b. Graph and label both on the same coordinate plane, and compare and contrast the two graphs.



*Both are decreasing exponentially and have the same y-intercept because they have the same initial temperature. The graph for Cup 2 has a larger vertical stretch than Cup 1, but Cup 1 has a larger vertical translation, which is why they both can have the same initial temperature. The y-values of Cup 2 level out lower than the corresponding y-values of Cup 1 because of the lower ambient temperature.*

*The temperature difference (between the cup and the surroundings) drives the cooling. Larger temperature differences lead to faster cooling. This is why the outdoor cup cools much faster.*

Ask students:

- Do these functions, graphs, and temperature values make sense? For example, if it is 42°F outside, can a cup of coffee ever cool to below 42°F?
  - No. The temperature of the cup will decrease until it is roughly the same temperature as its ambient surroundings, which the graph shows.

c. Coffee is safe to drink when its temperature is below 140°F. Estimate how much time elapses before each cup is safe to drink.

*Cup 1: Approximately 2 min.*

*Cup 2: Approximately 1.5 min.*

When discussing the results, ask students how they found the answer to part (c). Some may have used the graph; some may have plugged time (in number of minutes) into the formula using trial and error. Allow students to demonstrate both methods.

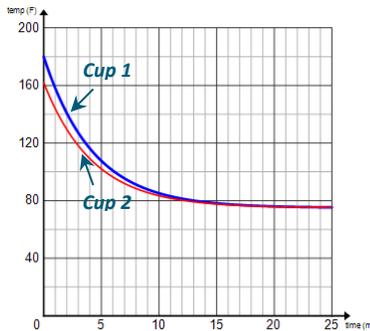
2. Suppose both cups are poured at the same time, and both are left sitting in the room that is 75°F. But this time, milk is immediately poured into Cup 2, cooling it to an initial temperature of 162°F.

a. Use Newton's law of cooling to write equations for the temperature of each cup of coffee after  $t$  minutes has elapsed.

*Cup 1:  $T_1(t) = 75 + (180 - 75) \cdot 2.718^{-0.2337t}$*

*Cup 2:  $T_2(t) = 75 + (162 - 75) \cdot 2.718^{-0.2337t}$*

- b. Graph and label both on the same coordinate plane, and compare and contrast the two graphs.



*Both are decreasing exponentially, and both appear to level out to approximately the same temperature (the room temperature). The graph for Cup 2 has a smaller vertical stretch than Cup 1 and a smaller y-intercept because of its lower starting temperature.*

- c. Coffee is safe to drink when its temperature is below 140°F. How much time elapses before each cup is safe to drink?

*Cup 1: Approximately 2 min.*

*Cup 2: Approximately 1 min.*

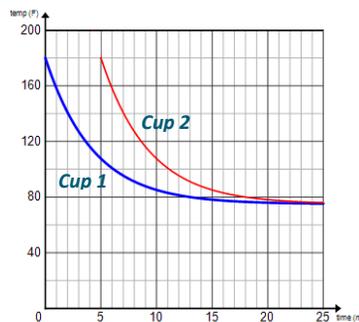
- Which should you do to drink the coffee sooner: Walk outside in the 42°F temperature, or pour milk into your coffee? Which scenario would have a greater impact on the temperature of the coffee over the long run?
  - *At first, decreasing the initial temperature with milk has the greater impact because the coffee cools enough to drink more quickly. But, as time elapses, decreasing the ambient temperature has the greater impact because the coffee continues to cool and levels out at a lower temperature.*

3. Suppose Cup 2 is poured 5 minutes after Cup 1 (the pot of coffee is maintained at 180°F over the 5 minutes). Both are left sitting in the room that is 75°F.

- a. Use the equation for Cup 1 found in Exercise 1, part (a) to write an equation for Cup 2.

*Cup 2:  $T(t) = 75 + (180 - 75) \cdot 2.718^{-0.2337(t-5)}$*

- b. Graph and label both on the same coordinate plane, and describe how to obtain the graph of Cup 2 from the graph of Cup 1.



*The graph of Cup 2 is the graph of Cup 1 shifted right 5 units.*

**Closing (5 minutes)**

- How does changing the initial coffee temperature affect the graph?
  - *The graph had a lower starting point (y-intercept). The cold milk cools the coffee quickly at first, but compared to the hotter cup, takes longer to cool to the same temperature. This is because the cooler cup has a smaller temperature difference with the ambient temperature, which leads to the slower cooling rate.*
- How does changing the ambient temperature of the coffee affect the graph?
  - *At a lower ambient temperature, the coffee cools more quickly and levels off to a lower temperature because of the cooler surrounding temperature.*

**Exit Ticket (5 minutes)**

Name \_\_\_\_\_

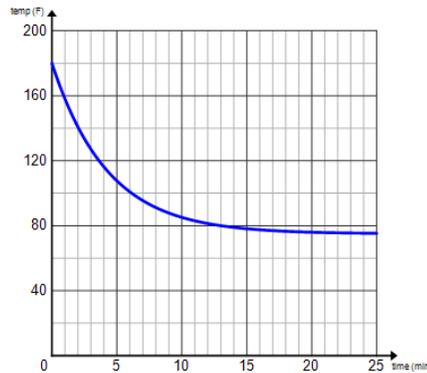
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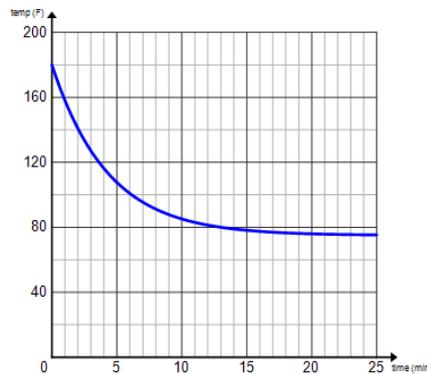
### Exit Ticket

Shown below is the graph of Cup 1 from the exercise completed in class. For each scenario, sketch and label a graph of Cup 2 on the same coordinate plane.

- Cup 2 is poured 10 minutes after Cup 1 (the pot of coffee is maintained at  $180^{\circ}\text{F}$  over the 10 minutes).



- Cup 2 is immediately taken outside where the temperature is  $90^{\circ}\text{F}$ .

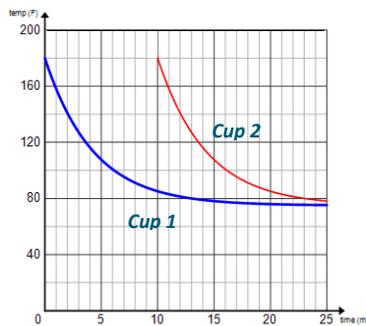


Exit Ticket Sample Solutions

Shown below is the graph of Cup 1 from the exercise completed in class. For each scenario, sketch and label a graph of Cup 2 on the same coordinate plane.

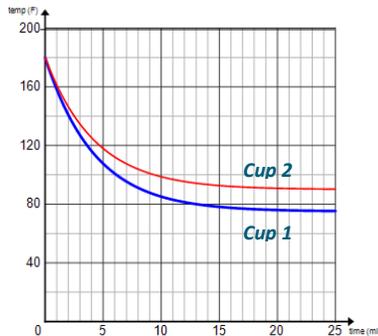
- Cup 2 is poured 10 minutes after Cup 1 (the pot of coffee is maintained at 180°F over the 10 minutes).

Answer:



- Cup 2 is immediately taken outside where the temperature is 90°F.

Answer:



Problem Set Sample Solutions

Use the Coffee Cooling demonstration on Wolfram Alpha to write a short report on the questions that follow.

<http://demonstrations.wolfram.com/TheCoffeeCoolingProblem/>

(Note that Wolfram’s free CDF player needs to be downloaded ahead of time in order to be able to run the demonstration.)

- If you want your coffee to become drinkable as quickly as possible, should you add cream immediately after pouring the coffee or wait? Use results from the demonstration to support your claim.

Answers will vary, but the results may surprise students. If you want to immediately cool the coffee, you should add cream. But if you are trying to reduce the temperature to 143°F (the temperature where it becomes drinkable according to the demonstration) as quickly as possible, you should wait to add the cream. For example, if you add cream to the first cup immediately and then wait 2 minutes (120 seconds) to add cream to the second cup, the second cup of reaches 143°F almost 30 seconds before the first cup does. Keeping the difference between the temperature of the coffee and the ambient temperature (the temperature of the surrounding air) larger for a longer interval of time causes the coffee to cool more quickly.

2. If you want your coffee to stay warm longer, should you add cream immediately after pouring the coffee or wait? Use results from the demonstration to support your claim.

*It turns out that if you want the coffee to stay hot as long as possible, then you should add the cream immediately. This will initially cool the temperature but then the rate at which the coffee is cooling will be less than the rate would be without the cream. Once the difference between the temperature of the coffee and the ambient temperature is smaller, the rate of cooling is slower, and the coffee stays warmer longer. For example, if you add cream immediately to one cup of coffee and wait 4 minutes (240 seconds) to add cream to the second cup, after 500 seconds the temperature of the first cup of coffee is  $2.2^{\circ}\text{F}$  greater than the temperature of the second cup.*