

Lesson 12: Decimal Expansions of Fractions, Part 2

Classwork

Example 1

Find the decimal expansion of $\frac{35}{11}$.

Exercises 1–3

1. Find the decimal expansion of $\frac{5}{3}$ without using long division.

2. Find the decimal expansion of $\frac{5}{11}$ without using long division.

3. Find the decimal expansion of the number $\frac{23}{99}$ first without using long division and then again using long division.

Lesson Summary

For rational numbers, there is no need to guess and check in which interval of tenths, hundredths, or thousandths the number will lie.

For example, to determine where the fraction $\frac{1}{8}$ lies in the interval of tenths, compute using the following inequality:

$$\begin{array}{ll} \frac{m}{10} < \frac{1}{8} < \frac{m+1}{10} & \text{Use the denominator of 10 because we need to find the tenths} \\ & \text{digit of } \frac{1}{8}. \\ m < \frac{10}{8} < m+1 & \text{Multiply through by 10.} \\ m < 1\frac{1}{4} < m+1 & \text{Simplify the fraction } \frac{10}{8}. \end{array}$$

The last inequality implies that $m = 1$ and $m + 1 = 2$ because $1 < 1\frac{1}{4} < 2$. Then, the tenths digit of the decimal expansion of $\frac{1}{8}$ is 1.

To find in which interval of hundredths $\frac{1}{8}$ lies, we seek consecutive integers m and $m + 1$ so that

$$\frac{1}{10} + \frac{m}{100} < \frac{1}{8} < \frac{1}{10} + \frac{m+1}{100}.$$

This is equivalent to

$$\frac{m}{100} < \frac{1}{8} - \frac{1}{10} < \frac{m+1}{100},$$

so we compute $\frac{1}{8} - \frac{1}{10} = \frac{2}{80} = \frac{1}{40}$. We have

$$\frac{m}{100} < \frac{1}{40} < \frac{m+1}{100}.$$

Multiplying through by 100 gives

$$m < \frac{10}{4} < m+1.$$

The last inequality implies that $m = 2$ and $m + 1 = 3$ because $2 < 2\frac{1}{2} < 3$. Then, the hundredths digit of the decimal expansion of $\frac{1}{8}$ is 2.

We can continue the process until the decimal expansion is complete or until we suspect a repeating pattern that we can verify.

Problem Set

1. Without using long division, explain why the tenths digit of $\frac{3}{11}$ is a 2.
2. Find the decimal expansion of $\frac{25}{9}$ without using long division.
3. Find the decimal expansion of $\frac{11}{41}$ to at least 5 digits without using long division.
4. Which number is larger, $\sqrt{10}$ or $\frac{28}{9}$? Answer this question without using long division.
5. Sam says that $\frac{7}{11} = 0.63$, and Jaylen says that $\frac{7}{11} = 0.636$. Who is correct? Why?