Lesson 11: The Decimal Expansion of Some Irrational Numbers

Classwork

Opening Exercise

Place $\sqrt{28}$ on a number line. Make a guess as to the first few values of the decimal expansion of $\sqrt{28}$. Explain your reasoning.

Example 1

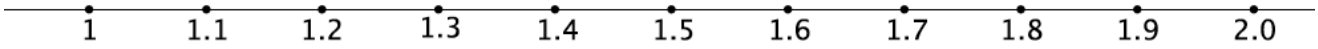
Consider the decimal expansion of $\sqrt{3}$.

Find the first two values of the decimal expansion using the following fact: If $c^2 < 3 < d^2$ for positive numbers c and d, then $c < \sqrt{3} < d$.

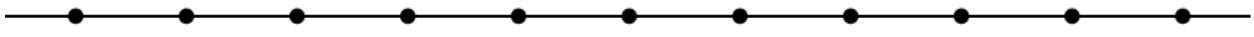
First Approximation:

Because $1 < 3 < 4$, we have $1 < \sqrt{3} < 2$.

Second approximation:



Third approximation:

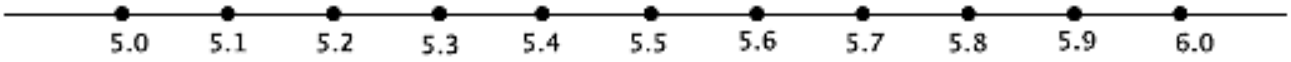


Example 2

Find the first few places of the decimal expansion of $\sqrt{28}$.

First approximation:

Second approximation:



Third approximation:

Fourth approximation:

Exercise 1

In which interval of hundredths does $\sqrt{14}$ lie? Show your work.

Lesson Summary

To find the first few decimal places of the decimal expansion of the square root of a non-perfect square, first determine between which two integers the square root lies, then in which interval of a tenth the square root lies, then in which interval of a hundredth it lies, and so on.

Example: Find the first few decimal places of $\sqrt{22}$.

Begin by determining between which two integers the number would lie.
$\sqrt{22}$ is between the integers 4 and 5 because $16 < 22 < 25$.

Next, determine between which interval of tenths the number belongs.
$\sqrt{22}$ is between 4.6 and 4.7 because $4.6^2 = 21.16 < 22 < 4.7^2 = 22.09$.

Next, determine between which interval of hundredths the number belongs.
$\sqrt{22}$ is between 4.69 and 4.70 because $4.69^2 = 21.9961 < 22 < 4.70^2 = 22.0900$.

A good estimate of the value of $\sqrt{22}$ is 4.69. It is correct to two decimal places and so has an error no larger than 0.01.

Notice that with each step of this process we are getting closer and closer to the actual value $\sqrt{22}$. This process can continue using intervals of thousandths, ten-thousandths, and so on.

Problem Set

1. In which hundredth interval of the number line does $\sqrt{84}$ lie?

2. Determine the three-decimal digit approximation of the number $\sqrt{34}$.

3. Write the decimal expansion of $\sqrt{47}$ to at least two-decimal digits.

4. Write the decimal expansion of $\sqrt{46}$ to at least two-decimal digits.

5. Explain how to improve the accuracy of the decimal expansion of an irrational number.

6. Is the number $\sqrt{144}$ rational or irrational? Explain.

7. Is the number $0.\overline{64} = 0.646464646\ldots$ rational or irrational? Explain.

8. Henri computed the first 100 decimal digits of the number $\frac{352}{541}$ and got

$$0.650646950092421441774491682070240295748613678373382624768946$$
$$39556377079482439926062846580406654343807763401109057301294\ldots.$$

He saw no repeating pattern to the decimal and so concluded that the number is irrational. Do you agree with Henri's conclusion? If not, what would you say to Henri?

9. Use a calculator to determine the decimal expansion of $\sqrt{35}$. Does the number appear to be rational or irrational? Explain.

10. Use a calculator to determine the decimal expansion of $\sqrt{101}$. Does the number appear to be rational or irrational? Explain.

11. Use a calculator to determine the decimal expansion of $\sqrt{7}$. Does the number appear to be rational or irrational? Explain.

12. Use a calculator to determine the decimal expansion of $\sqrt{8720}$. Does the number appear to be rational or irrational? Explain.

13. Use a calculator to determine the decimal expansion of $\sqrt{17956}$. Does the number appear to be rational or irrational? Explain.

14. Since the number $\frac{3}{5}$ is rational, must the number $\left(\frac{3}{5}\right)^2$ be rational as well? Explain.

15. If a number x is rational, must the number x^2 be rational as well? Explain.

16. Challenge: Determine the two-decimal digit approximation of the number $\sqrt[3]{9}$.