Lesson 2: Square Roots

Student Outcomes

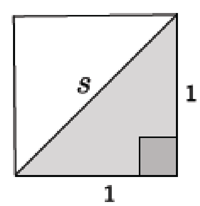
- Students are introduced to the notation for square roots.
- Students approximate the location of square roots of whole numbers on the number line.

Classwork

Discussion (10 minutes)

MP.1 As an option, the discussion can be framed as a challenge. Distribute compasses, and ask students, "How can we determine an estimate for the length of the diagonal of the unit square?"

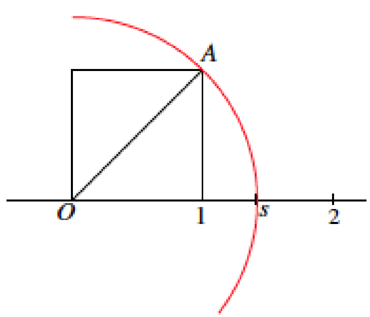
- Consider a unit square, a square with side lengths equal to 1. How can we determine the length of the diagonal, s, of the unit square?



  - *We can use the Pythagorean theorem to determine the length of the diagonal.*

$$1^2 + 1^2 = s^2$$
$$2 = s^2$$

- What number, s, times itself is equal to 2?
  - *We don't know exactly, but we know the number has to be between* 1 *and* 2.
- We can show that the number must be between 1 and 2 if we place the unit square on a number line as shown. Then, consider a circle with center O and radius equal to the length of the hypotenuse, segment OA, of the triangle.



> *Scaffolding:*
> Depending on students' experience, it may be useful to review or teach the concept of square numbers and perfect squares.

- We can see that length OA is somewhere between 1 and 2 but precisely at point s. But what is that number s?

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- From our work with exponents, specifically squared numbers, we know that 2 is not a perfect square. Thus, the length of the diagonal must be between the two integers 1 and 2, and that is confirmed on the number line. To determine the number s, we should look at that part of the number line more closely. To do so, we need to discuss what kinds of numbers lie between the integers on a number line. What do we already know about those numbers?

Lead a discussion about the types of numbers found between the integers on a number line. Students should identify that rational numbers, such as fractions and decimals, lie between the integers. Have students give concrete examples of numbers found between the integers 1 and 2. Consider asking students to write a rational number, x, so that $1 < x < 2$, on a sticky note and then to place it on a number line drawn on a poster or white board. At the end of this part of the discussion, make clear that all of the numbers students identified are rational and in the familiar forms of fractions, mixed numbers, and decimals. Then, continue with the discussion below about square roots.

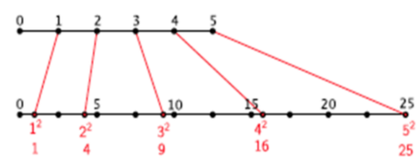
- There are other numbers on the number line between the integers. Some of the square roots of whole numbers are equal to whole numbers, but most lie between the integers on the number line. A positive number whose square is equal to a positive number b is denoted by the symbol $\sqrt{b}$. The symbol $\sqrt{b}$ automatically denotes a positive number (e.g., $\sqrt{4}$ is always 2, not -2). The number $\sqrt{b}$ is called *a positive square root of* b. We will soon learn that it is *the* positive square root (i.e., there is only one).
- What is $\sqrt{25}$, that is, the positive square root of 25? Explain.
  - *The positive square root of* 25 *is* 5 *because* $5^2 = 25$.
- What is $\sqrt{9}$, that is, the positive square root of 9? Explain.
  - *The positive square root of* 9 *is* 3 *because* $3^2 = 9$.

Scaffolding:

Students may benefit from an oral recitation of square roots of perfect squares here and throughout the module. Consider some repeated *quick practice*, calling out, for example: “What is the square root of 81?” and “What is the square root of 100?” Ask for choral or individual responses.

Scaffolding:

If students are struggling with the concept of a square root, it may help to refer to visuals that relate numbers and their squares. Showing this visual



and asking questions (e.g., “What is the square root of 9?”) builds students’ understanding of square roots through their understanding of squares.

Exercises 1–4 (5 minutes)

Students complete Exercises 1–4 independently.

Exercises 1–4

1. **Determine the positive square root of 81, if it exists. Explain.**

   The square root of 81 is 9 because $9^2 = 81$.

2. **Determine the positive square root of 225, if it exists. Explain.**

   The square root of 225 is 15 because $15^2 = 225$.

3. **Determine the positive square root of -36, if it exists. Explain.**

   The number -36 does not have a square root because there is no number squared that can produce a negative number.

4. **Determine the positive square root of 49, if it exists. Explain.**

   The square root of 49 is 7 because $7^2 = 49$.

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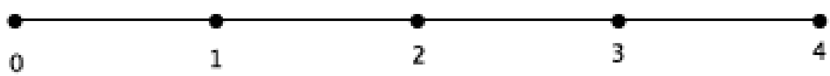
Discussion (15 minutes)

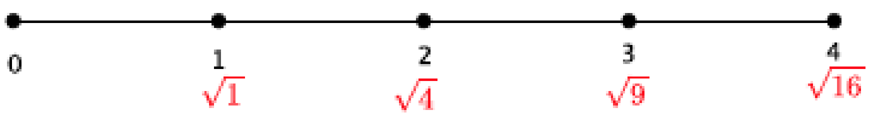
- Now, back to our unit square. We said that the length of the diagonal is s, and $s^2 = 2$. Now that we know about square roots, we can say that the length of s is $\sqrt{2}$ and that the number $\sqrt{2}$ is between integers 1 and 2. Let's look at the number line more generally to see if we can estimate the value of $\sqrt{2}$.
- Take a number line from 0 to 4:

Discussion



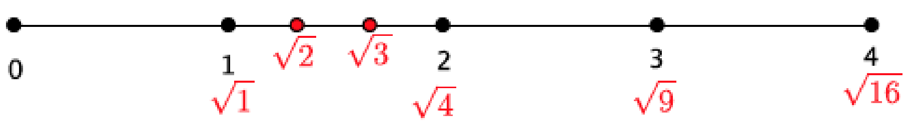
- Place the numbers $\sqrt{1}$, $\sqrt{4}$, $\sqrt{9}$, and $\sqrt{16}$ on the number line, and explain how you knew where to place them.

Solutions are shown below in red.



- Place the numbers $\sqrt{2}$ and $\sqrt{3}$ on the number line. Be prepared to explain your reasoning.

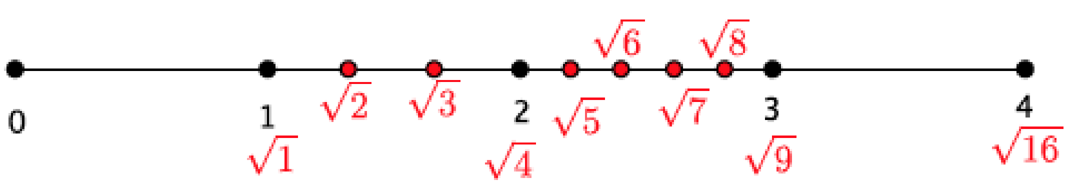
Solutions are shown below in red. Students should reason that the numbers $\sqrt{2}$ and $\sqrt{3}$ belong on the number line between $\sqrt{1}$ and $\sqrt{4}$. They might be more specific and suggest that the numbers $\sqrt{2}$ and $\sqrt{3}$ sit equally spaced in the interval between 1 and 2. This idea suggests that $1\frac{1}{3}$ might be a good approximation for $\sqrt{2}$ and $1\frac{2}{3}$ for $\sqrt{3}$. Of course, this suggested spacing is just speculation for now.



MP.3

- Place the numbers $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, and $\sqrt{8}$ on the number line. Be prepared to explain your reasoning.

Solutions are shown below in red. The discussion about placement should be similar to the previous one.



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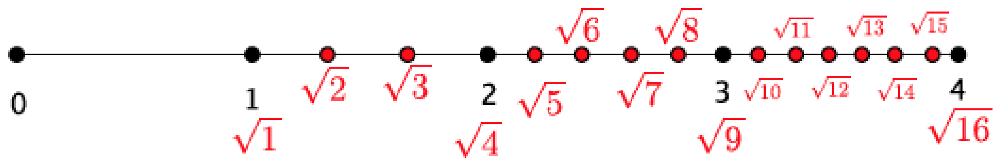
MP.3

- Place the numbers $\sqrt{10}$, $\sqrt{11}$, $\sqrt{12}$, $\sqrt{13}$, $\sqrt{14}$, and $\sqrt{15}$ on the number line. Be prepared to explain your reasoning.

Solutions are shown below in red. The discussion about placement should be similar to the previous one.



- Our work on the number line shows that there are many more square roots of whole numbers that are not perfect squares than those that are perfect squares. On the number line above, we have four perfect square numbers and twelve that are not! After we do some more work with roots, in general, we will cover exactly how to describe these numbers and how to approximate their values with greater precision. For now, we will estimate their locations on the number line using what we know about perfect squares.

Exercises 5–9 (5 minutes)

Students complete Exercises 5–9 independently. Calculators may be used for approximations.

Exercises 5–9

Determine the positive square root of the number given. If the number is not a perfect square, determine which whole number the square root would be closest to, and then use *guess and check* to give an approximate answer to one or two decimal places.

5. $\sqrt{49}$

7

6. $\sqrt{62}$

The square root of 62 ***is close to*** 8***. The square root of*** 62 ***is approximately*** 7.9 ***because*** $7.9^2 = 62.41$.

7. $\sqrt{122}$

The square root of 122 ***is close to*** 11***. Students may guess a number between*** 11 ***and*** 11.1 ***because*** $11.05^2 = 122.1025$.

8. $\sqrt{400}$

20

9. Which of the numbers in Exercises 5–8 are not perfect squares? Explain.

The numbers 62 ***and*** 122 ***are not perfect squares because there is no integer*** x ***to satisfy*** $x^2 = 62$ ***or*** $x^2 = 122$.

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Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know that there are numbers on the number line between the integers. The ones we looked at in this lesson are square roots of whole numbers that are not perfect squares.
- We know that when a positive number x is squared and the result is b, then $\sqrt{b}$ is equal to x.
- We know how to approximate the square root of a whole number and its location on a number line by figuring out which two perfect squares it is between.

> **Lesson Summary**
>
> **A positive number whose square is equal to a positive number b is denoted by the symbol $\sqrt{b}$. The symbol $\sqrt{b}$ automatically denotes a positive number. For example, $\sqrt{4}$ is always 2, not -2. The number $\sqrt{b}$ is called *a positive square root of b*.**
>
> **The square root of a perfect square of a whole number is that whole number. However, there are many whole numbers that are not perfect squares.**

Exit Ticket (5 minutes)

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Name ______________________________ Date ______________

Lesson 2: Square Roots

Exit Ticket

1. Write the positive square root of a number x in symbolic notation.

2. Determine the positive square root of 196. Explain.

3. The positive square root of 50 is not an integer. Which whole number does the value of $\sqrt{50}$ lie closest to? Explain.

4. Place the following numbers on the number line in approximately the correct positions: $\sqrt{16}$, $\sqrt{9}$, $\sqrt{11}$, and 3.5.

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Exit Ticket Sample Solutions

1. **Write the square root of a number x in symbolic notation.**

   $\sqrt{x}$

2. **Determine the positive square root of 196. Explain.**

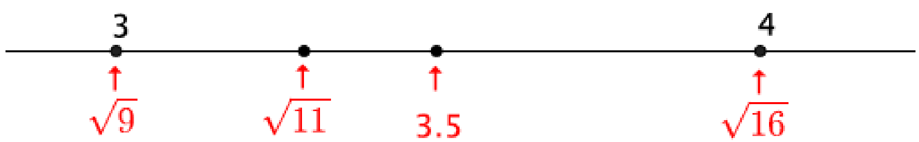
   $\sqrt{196} = 14$ because $14^2 = 196$.

3. **The positive square root of 50 is not an integer. Which whole number does the value $\sqrt{50}$ lie closest to? Explain.**

   $\sqrt{50}$ is between 7 and 8 but closer to 7. The reason is that $7^2 = 49$, and $8^2 = 64$. The number 50 is between 49 and 64 but closer to 49. Therefore, the square root of 50 is close to 7.

4. **Place the following numbers on the number line in approximately the correct positions: $\sqrt{16}$, $\sqrt{9}$, $\sqrt{11}$, and 3.5.**

   Solutions are shown in red below.



Problem Set Sample Solutions

Determine the positive square root of the number given. If the number is not a perfect square, determine the integer to which the square root would be closest.

1. $\sqrt{169}$

   13

2. $\sqrt{256}$

   16

3. $\sqrt{81}$

   9

4. $\sqrt{147}$

   The number 147 is not a perfect square. It is between the perfect squares 144 and 169 but closer to 144. Therefore, the square root of 147 is close to 12.

5. $\sqrt{8}$

   The number 8 is not a perfect square. It is between the perfect squares 4 and 9 but closer to 9. Therefore, the square root of 8 is close to 3.

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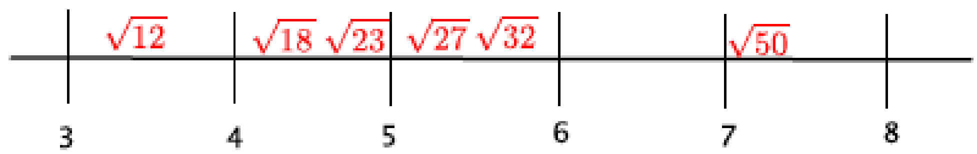
6. **Which of the numbers in Problems 1–5 are not perfect squares? Explain.**

   The numbers 147 and 8 are not perfect squares because there is no integer x so that $x^2 = 147$ or $x^2 = 8$.

7. **Place the following list of numbers in their approximate locations on a number line.**

   $\sqrt{32}$, $\sqrt{12}$, $\sqrt{27}$, $\sqrt{18}$, $\sqrt{23}$, and $\sqrt{50}$



   Answers are noted in red.

8. **Between which two integers will $\sqrt{45}$ be located? Explain how you know.**

   The number 45 is not a perfect square. It is between the perfect squares 36 and 49 but closer to 49. Therefore, the square root of 45 is between the integers 6 and 7 because $\sqrt{36} = 6$ and $\sqrt{49} = 7$ and $\sqrt{36} < \sqrt{45} < \sqrt{49}$.

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