Lesson 8: Graphs of Simple Nonlinear Functions

Student Outcomes

- Students examine the average rate of change for nonlinear function over various intervals and verify that these values are not constant.

Lesson Notes

In Exercises 4–10, students are given the option to sketch the graphs of given equations to verify their claims about them being linear or nonlinear. For this reason, students may need graph paper to complete these exercises. Students need graph paper to complete the Problem Set.

Classwork

Exploratory Challenge/Exercises 1–3 (19 minutes)

Students work independently or in pairs to complete Exercises 1–3.

Exploratory Challenge/Exercises 1–3

1. **Consider the function that assigns to each number x the value x^2.**

   a. **Do you think the function is linear or nonlinear? Explain.**

   I think the function is nonlinear. The equation describing the function is not of the form $y = mx + b$.

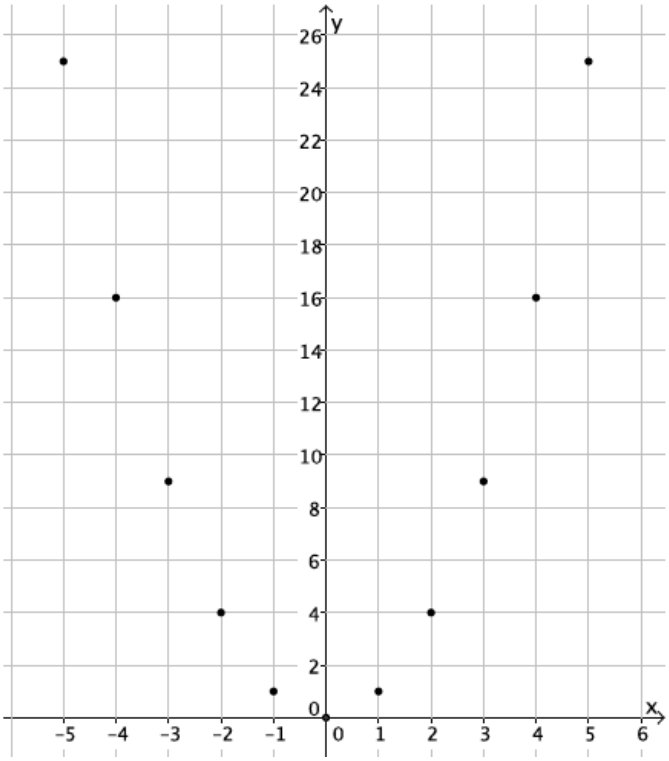
   b. **Develop a list of inputs and outputs for this function. Organize your work using the table below. Then, answer the questions that follow.**

Input (x)	Output (x^2)
-5	25
-4	16
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9
4	16
5	25

Scaffolding:

Students may benefit from exploring these exercises in small groups.

c. Plot the inputs and outputs as ordered pairs defining points on the coordinate plane.



d. What shape does the graph of the points appear to take?

It appears to take the shape of a curve.

e. Find the rate of change using rows 1 and 2 from the table above.

$$\frac{25-16}{-5-(-4)}=\frac{9}{-1}=-9$$

f. Find the rate of change using rows 2 and 3 from the table above.

$$\frac{16-9}{-4-(-3)}=\frac{7}{-1}=-7$$

g. Find the rate of change using any two other rows from the table above.

Student work will vary.

$$\frac{16-25}{4-5}=\frac{-9}{-1}=9$$

h. Return to your initial claim about the function. Is it linear or nonlinear? Justify your answer with as many pieces of evidence as possible.

This is definitely a nonlinear function because the rate of change is not a constant for different intervals of inputs. Also, we would expect the graph of a linear function to be a set of points in a line, and this graph is not a line. As was stated before, the expression x^2 is nonlinear.

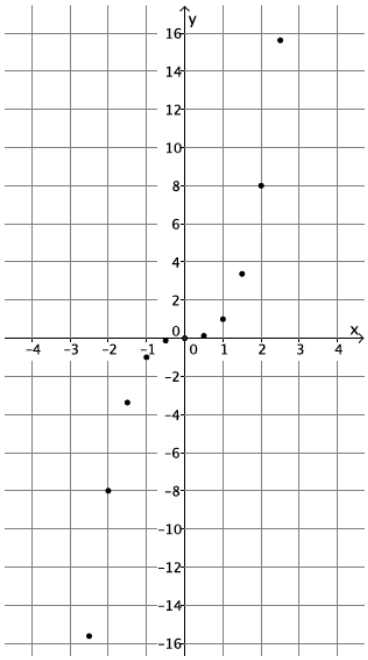
2. Consider the function that assigns to each number x the value x^3.

a. Do you think the function is linear or nonlinear? Explain.

I think the function is nonlinear. The equation describing the function is not of the form $y = mx + b$.

b. Develop a list of inputs and outputs for this function. Organize your work using the table below. Then, answer the questions that follow.

Input (x)	Output (x^3)
-2.5	-15.625
-2	-8
-1.5	-3.375
-1	-1
-0.5	-0.125
0	0
0.5	0.125
1	1
1.5	3.375
2	8
2.5	15.625



c. Plot the inputs and outputs as ordered pairs defining points on the coordinate plane.

d. What shape does the graph of the points appear to take?

It appears to take the shape of a curve.

e. Find the rate of change using rows 2 and 3 from the table above.

$$\frac{-8-(-3.375)}{-2-(-1.5)} = \frac{-4.625}{-0.5} = 9.25$$

f. Find the rate of change using rows 3 and 4 from the table above.

$$\frac{-3.375-(-1)}{-1.5-(-1)} = \frac{-2.375}{-0.5} = 4.75$$

g. Find the rate of change using rows 8 and 9 from the table above.

$$\frac{1-3.375}{1-1.5} = \frac{-2.375}{-0.5} = 4.75$$

h. Return to your initial claim about the function. Is it linear or nonlinear? Justify your answer with as many pieces of evidence as possible.

This is definitely a nonlinear function because the rate of change is not a constant for any interval of inputs. Also, we would expect the graph of a linear function to be a line, and this graph is not a line. As was stated before, the expression x^3 is nonlinear.

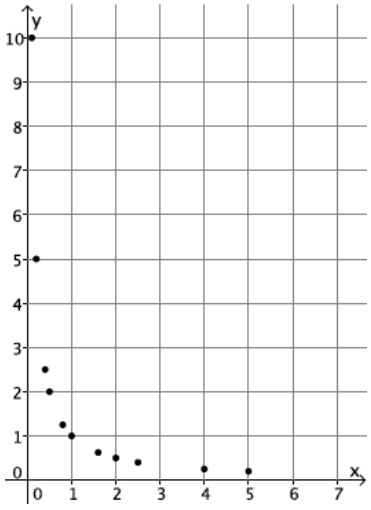
3. Consider the function that assigns to each positive number x the value $\frac{1}{x}$.

a. Do you think the function is linear or nonlinear? Explain.

I think the function is nonlinear. The equation describing the function is not of the form $y = mx + b$.

b. Develop a list of inputs and outputs for this function. Organize your work using the table below. Then, answer the questions that follow.

Input (x)	Output $\left(\frac{1}{x}\right)$
0.1	10
0.2	5
0.4	2.5
0.5	2
0.8	1.25
1	1
1.6	0.625
2	0.5
2.5	0.4
4	0.25
5	0.2



c. Plot the inputs and outputs as ordered pairs defining points on the coordinate plane.

d. What shape does the graph of the points appear to take?

It appears to take the shape of a curve.

e. Find the rate of change using rows 1 and 2 from the table above.

$$\frac{10-5}{0.1-0.2} = \frac{5}{-0.1} = -50$$

f. Find the rate of change using rows 2 and 3 from the table above.

$$\frac{5-2.5}{0.2-0.4} = \frac{2.5}{-0.2} = -12.5$$

g. Find the rate of change using any two other rows from the table above.

Student work will vary.

$$\frac{1-0.625}{1-1.6} = \frac{0.375}{-0.6} = -0.625$$

h. Return to your initial claim about the function. Is it linear or nonlinear? Justify your answer with as many pieces of evidence as possible.

This is definitely a nonlinear function because the rate of change is not a constant for any interval of inputs. Also, we would expect the graph of a linear function to be a line, and this graph is not a line. As was stated before, the expression $\frac{1}{x}$ is nonlinear.

Discussion (4 minutes)

- What did you notice about the rates of change in the preceding three problems?
  - *The rates of change were not constant in each of the three problems.*
- If the rate of change for pairs of inputs and corresponding outputs were the same for each and every pair, then what can we say about the function?
  - *We know the function is linear.*
- If the rate of change for pairs of inputs and corresponding outputs is not the same for each pair, what can we say about the function?
  - *We know the function is nonlinear.*
- Recall that any linear function can be described by an equation of the form $y = mx + b$. Any equation that cannot be written in this form is not linear, and its corresponding function is nonlinear.

Exercises 4–10 (12 minutes)

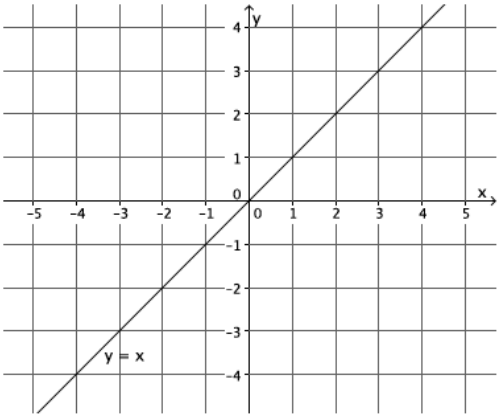
Students work independently or in pairs to complete Exercises 4–10.

Exercises 4–10

In each of Exercises 4–10, an equation describing a rule for a function is given, and a question is asked about it. If necessary, use a table to organize pairs of inputs and outputs, and then plot each on a coordinate plane to help answer the question.

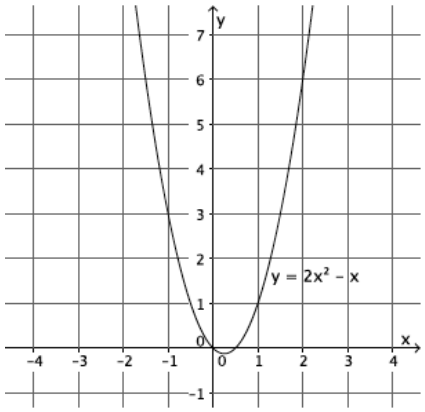
4. What shape do you expect the graph of the function described by $y = x$ to take? Is it a linear or nonlinear function?

I expect the shape of the graph to be a line. This function is a linear function described by the linear equation $y = x$. The graph of this function is a line.

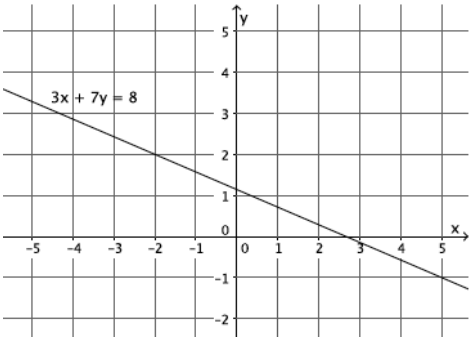


5. What shape do you expect the graph of the function described by $y = 2x^2 - x$ to take? Is it a linear or nonlinear function?

I expect the shape of the graph to be something other than a line. This function is nonlinear because its graph is not a line. Also the equation describing the function is not of the form $y = mx + b$. It is not linear.

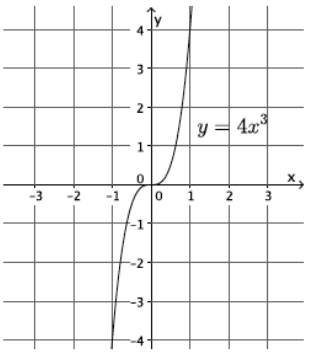


6. **What shape do you expect the graph of the function described by $3x + 7y = 8$ to take? Is it a linear or nonlinear function?**



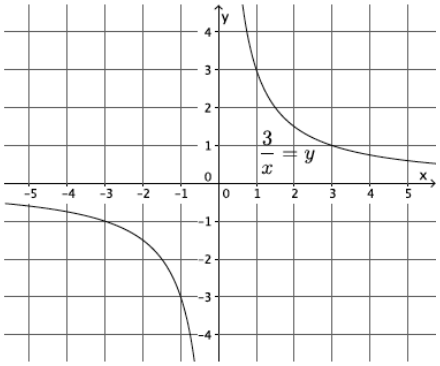
I expect the shape of the graph to be a line. This function is a linear function described by the linear equation $3x + 7y = 8$. The graph of this function is a line. (We have $y = -\frac{3}{7}x + \frac{8}{7}$.)

7. **What shape do you expect the graph of the function described by $y = 4x^3$ to take? Is it a linear or nonlinear function?**



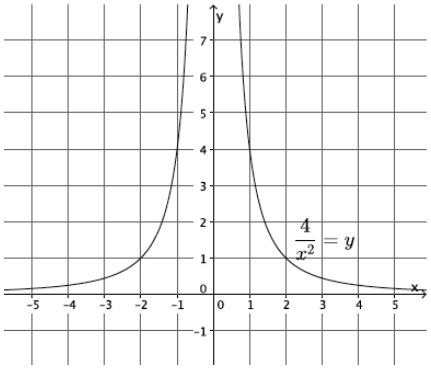
I expect the shape of the graph to be something other than a line. This function is nonlinear because its graph is not a line. Also the equation describing the function is not of the form $y = mx + b$. It is not linear.

8. **What shape do you expect the graph of the function described by $\frac{3}{x} = y$ to take? Is it a linear or nonlinear function? (Assume that an input of $x = 0$ is disallowed.)**



I expect the shape of the graph to be something other than a line. This function is nonlinear because its graph is not a line. Also the equation describing the function is not of the form $y = mx + b$. It is not linear.

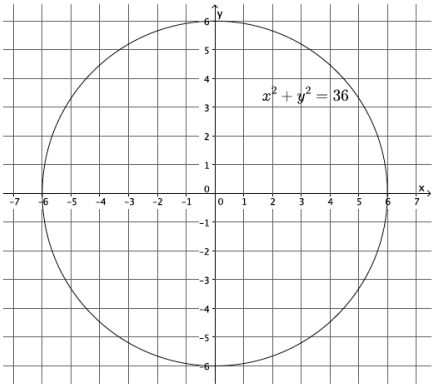
9. **What shape do you expect the graph of the function described by $\frac{4}{x^2} = y$ to take? Is it a linear or nonlinear function? (Assume that an input of $x = 0$ is disallowed.)**



I expect the shape of the graph to be something other than a line. This function is nonlinear because its graph is not a line. Also the equation describing the function is not of the form $y = mx + b$. It is not linear.

10. **What shape do you expect the graph of the equation $x^2 + y^2 = 36$ to take? Is it a linear or nonlinear function? Is it a function? Explain.**

I expect the shape of the graph to be something other than a line. It is nonlinear because its graph is not a line. It is not a function because there is more than one output for any given value of x in the interval $(-6, 6)$. For example, at $x = 0$ the y-value is both 6 and -6. This does not fit the definition of function because functions assign to each input exactly one output. Since there is at least one instance where an input has two outputs, it is not a function.



Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson.

- Students understand that, unlike linear functions, nonlinear functions do not have a constant rate of change.
- Students expect the graph of nonlinear functions to be some sort of curve.

Lesson Summary

One way to determine if a function is linear or nonlinear is to inspect average rates of change using a table of values. If these average rates of change are not constant, then the function is not linear.

Another way is to examine the graph of the function. If all the points on the graph do not lie on a common line, then the function is not linear.

If a function is described by an equation different from one equivalent to $y = mx + b$ for some fixed values m and b, then the function is not linear.

Exit Ticket (5 minutes)

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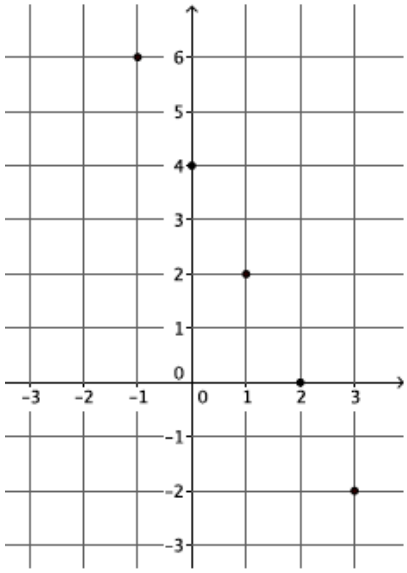
Lesson 8: Graphs of Simple Nonlinear Functions

Exit Ticket

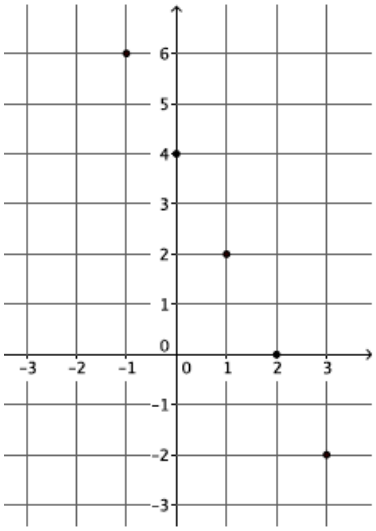
1. The graph below is the graph of a function. Do you think the function is linear or nonlinear? Briefly justify your answer.



2. Consider the function that assigns to each number x the value $\frac{1}{2}x^2$. Do you expect the graph of this function to be a straight line? Briefly justify your answer.

Exit Ticket Sample Solutions

1. **The graph below is the graph of a function. Do you think the function is linear or nonlinear? Briefly justify your answer.**



Student work may vary. The plot of this graph appears to be a straight line, and so the function is linear.

2. **Consider the function that assigns to each number x the value $\frac{1}{2}x^2$. Do you expect the graph of this function to be a straight line? Briefly justify your answer.**

The equation is nonlinear (not of the form $y = mx + b$), so the function is nonlinear. Its graph will not be a straight line.

Problem Set Sample Solutions

1. **Consider the function that assigns to each number x the value $x^2 - 4$.**

   a. **Do you think the function is linear or nonlinear? Explain.**

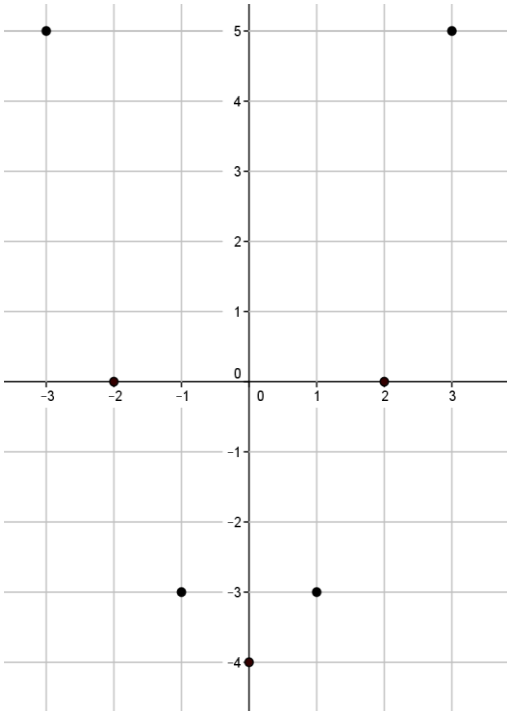
   The equation describing the function is not of the form $y = mx + b$. It is not linear.

   b. **Do you expect the graph of this function to be a straight line?**

   No

c. Develop a list of inputs and matching outputs for this function. Use them to begin a graph of the function.

Input (x)	Output $(x^2 - 4)$
-3	5
-2	0
-1	-3
0	-4
1	-3
2	0
3	5



d. Was your prediction to (b) correct?

Yes, the graph appears to be taking the shape of some type of curve.

2. Consider the function that assigns to each number x greater than -3 the value $\frac{1}{x+3}$.

a. Is the function linear or nonlinear? Explain.

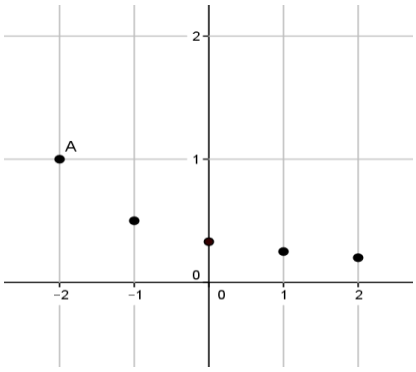
The equation describing the function is not of the form $y = mx + b$. It is not linear.

b. Do you expect the graph of this function to be a straight line?

No

c. Develop a list of inputs and matching outputs for this function. Use them to begin a graph of the function.

Input (x)	Output $\left(\frac{1}{x+3}\right)$
-2	1
-1	0.5
0	$0.3333\ldots$
1	0.25
2	0.2
3	$0.16666\ldots$



d. Was your prediction to (b) correct?

Yes, the graph appears to be taking the shape of some type of curve.

3.

a. **Is the function represented by this graph linear or nonlinear? Briefly justify your answer.**

The graph is clearly not a straight line, so the function is not linear.

b. **What is the average rate of change for this function from an input of $x = -2$ to an input of $x = -1$?**

$$\frac{-2-1}{-2-(-1)} = \frac{-3}{-1} = 3$$

c. **What is the average rate of change for this function from an input of $x = -1$ to an input of $x = 0$?**

$$\frac{1-2}{-1-0} = \frac{-1}{-1} = 1$$

As expected, the average rate of change of this function is not constant.