



Lesson 20: Every Line Is a Graph of a Linear Equation

Student Outcomes

- Students know that any non-vertical line is the graph of a linear equation in the form of $y = mx + b$, where b is a constant.
- Students write the linear equation whose graph is a given line.

Lesson Notes

The proof that every line is the graph of a linear equation in the Discussion is optional. If using the Discussion, skip the Opening Exercise, and resume the lesson with Example 1. Complete all other examples and exercises that follow. As an alternative to the Discussion, complete the Opening Exercise by showing a graph of a line on the coordinate plane and having students attempt to name the equation of the line. Two graphs are provided beginning on page 317. Have students write their equations and strategies for determining the equation of the line; then, lead the discussion described on page 317. Once students complete the Opening Exercise, work through Example 1 and the remaining examples and exercises in the lesson. Revisit the equations and strategies students developed by having them critique their reasoning in comparison to the work in the example; then, continue with the remainder of the lesson.

Classwork

Discussion (10 minutes)

- Now that we are confident that the graph of every linear equation is a line, can we say that every line is the graph of a linear equation? We can say yes with respect to vertical and horizontal lines; recall $x = c$ and $y = c$. But what about other non-vertical lines?
- We must prove that any non-vertical (and non-horizontal) line is a graph of a linear equation in the form of $y = mx + b$, where $m \neq 0$ and b are constants.
- Let l be any non-vertical (and non-horizontal) line. Suppose the slope of the line is m and that the line intersects the y -axis at point $Q(0, b)$.
- First, we show that any point on the line l is a point on the graph of the linear equation $y = mx + b$.
- Let $P(x, y)$ be any point on line l . We need to show that P is a solution to $y = mx + b$. Think about how we did this in the last lesson. What should we do?
 - Use the points P and Q in the slope formula.

$$m = \frac{y - b}{x - 0}$$

$$mx = y - b$$

$$mx + b = y - b + b$$

$$mx + b = y$$

- That shows that point P is a point on the graph of $y = mx + b$. Point Q is also on the graph of $y = mx + b$ because $b = m \cdot 0 + b$. Therefore, any point on the line l is a point on the graph of the linear equation $y = mx + b$.

MP.3

MP.3

- Now we want to show that any point on the graph of $y = mx + b$ is on l .
- Let R be any point on the graph of the linear equation $y = mx + b$. We know that the graph of $y = mx + b$ is a line with slope m . Let's call this line l' . We know that Q is on l' because $b = m \cdot 0 + b$. Therefore, l' is a line with slope m that passes through point Q . However, l is a line with slope m that passes through point Q . What does that mean about lines l and l' ?
 - *The lines l and l' are the same line because there is only one line with a given slope that can go through a given point.*
- Therefore, R is a point on l .
- Now we can be certain that every line is a graph of a linear equation.

Opening Exercise (10 minutes)

Show students Figure 1 below, and challenge them to write the equation for the line. Provide students time to work independently and then in pairs. Lead a discussion where students share their strategies for developing the equation of the line. Ask students how they knew their equations were correct; that is, did they verify that the points with integer coordinates were solutions to the equations they wrote? Ask students what kind of equation they wrote: linear or nonlinear. Ask students if they were given another line, could they write an equation for it using their strategy. Show them Figure 2 and, again, ask them to write the equation of the line. Verify that they wrote the correct equation, and conclude the discussion by stating that every line is the graph of a linear equation.

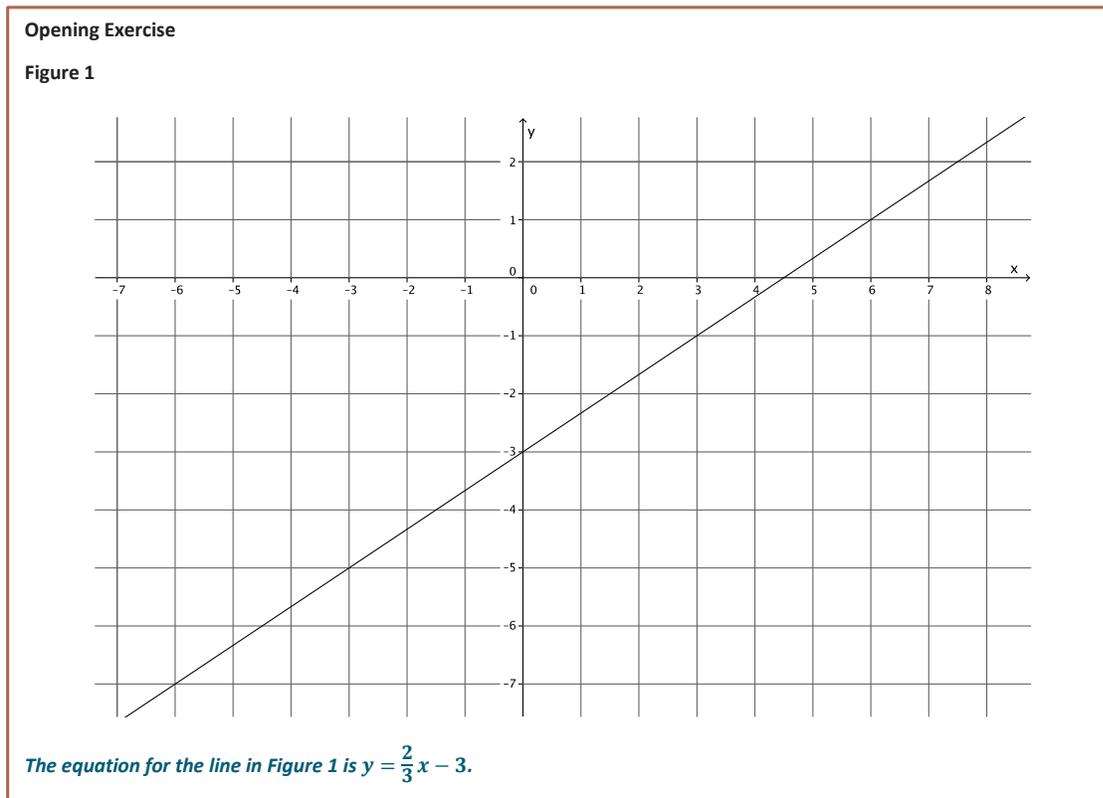
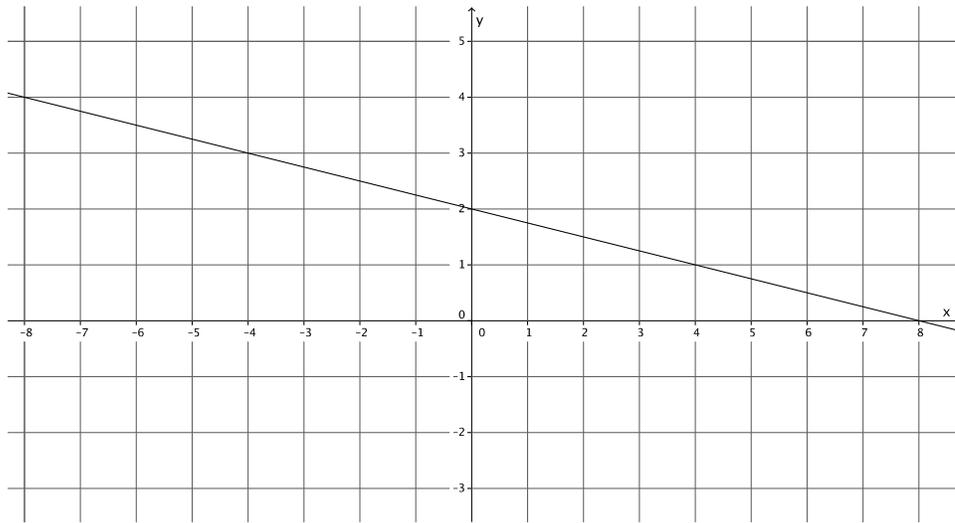


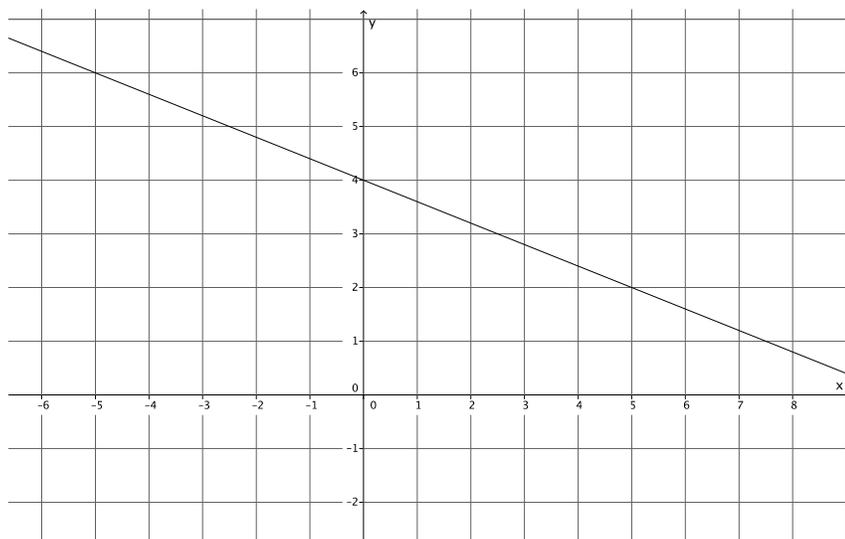
Figure 2



The equation for the line in Figure 2 is $y = -\frac{1}{4}x + 2$.

Example 1 (5 minutes)

- Given a line, we want to be able to write the equation that represents it.
- Which form of a linear equation do you think will be most valuable for this task: the standard form $ax + by = c$, or the slope-intercept form $y = mx + b$?
 - *The slope-intercept form because we can easily identify the slope and y-intercept point from both the equation and the graph.*
- Write the equation that represents the line shown below.

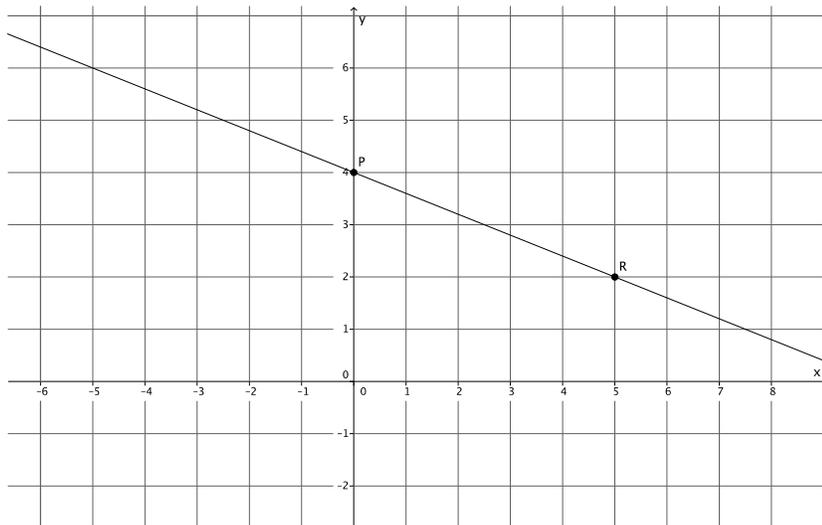


- First, identify the y -intercept point.
 - *The line intersects the y -axis at $(0, 4)$.*
- Now we must use what we know about slope to determine the slope of the line. Recall the following:

$$m = \frac{|QR|}{|PQ|}$$

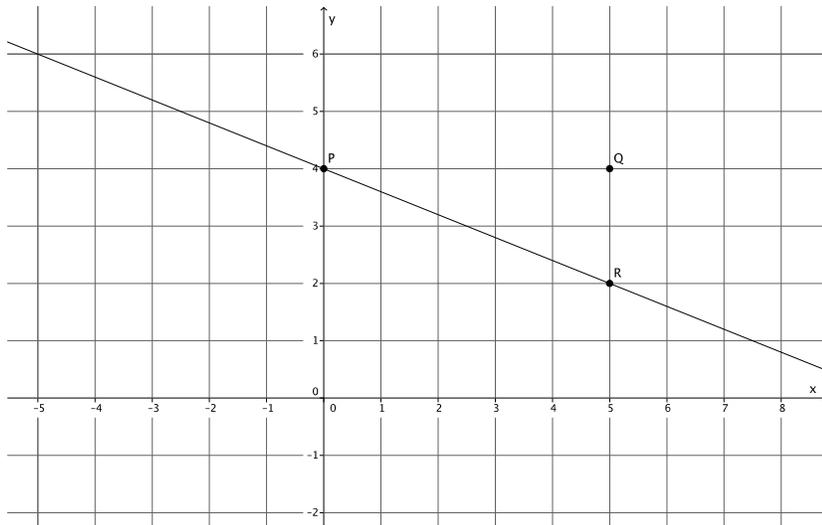
The point P represents our y -intercept point. Let's locate a point R on the line with integer coordinates.

- *We can use the point $(5, 2)$ or $(-5, 6)$.*
- We can use either point. For this example, let's use $(5, 2)$.

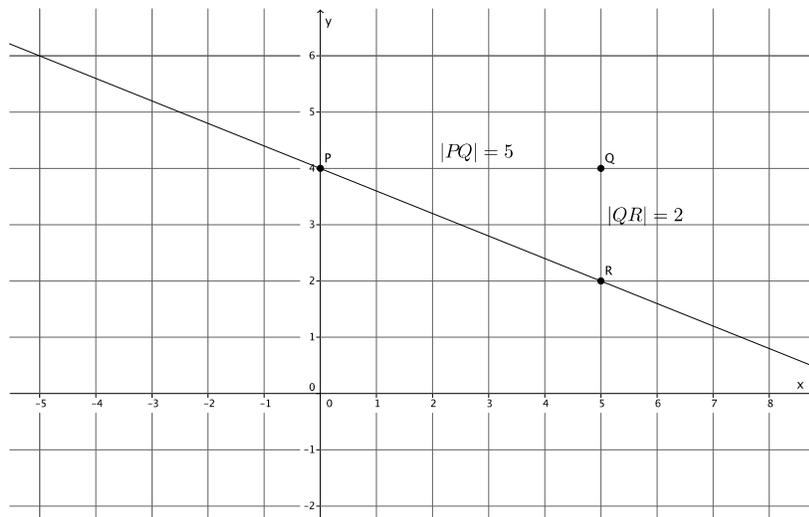


- Now we can locate point Q . It must be to the right of point P and be on a line parallel to the y -axis that goes through point R . What is the location of Q ?
 - *Point Q must be $(5, 4)$.*

- What fraction represents the slope of this line?



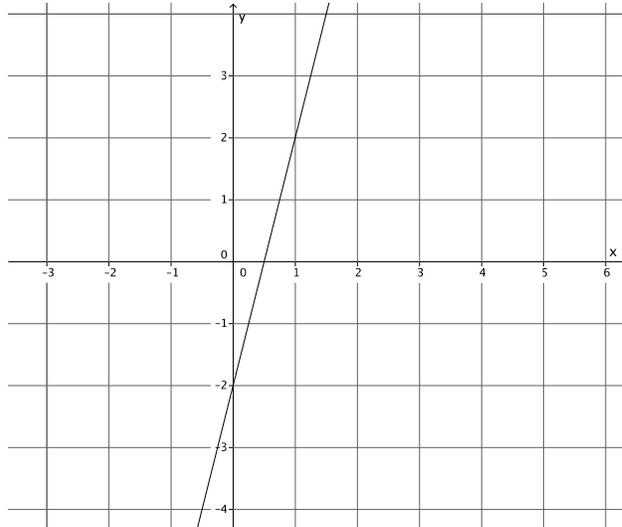
- The slope of the line is $m = -\frac{2}{5}$.
- The slope of the line is $m = -\frac{2}{5}$, and the y -intercept point is $(0, 4)$. What must the equation of the line be?



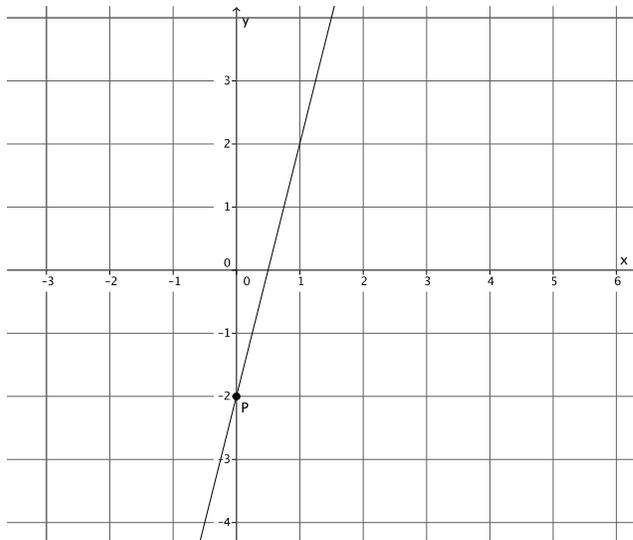
- The line is the graph of $y = -\frac{2}{5}x + 4$.

Example 2 (5 minutes)

- What is the y -intercept point of the line?

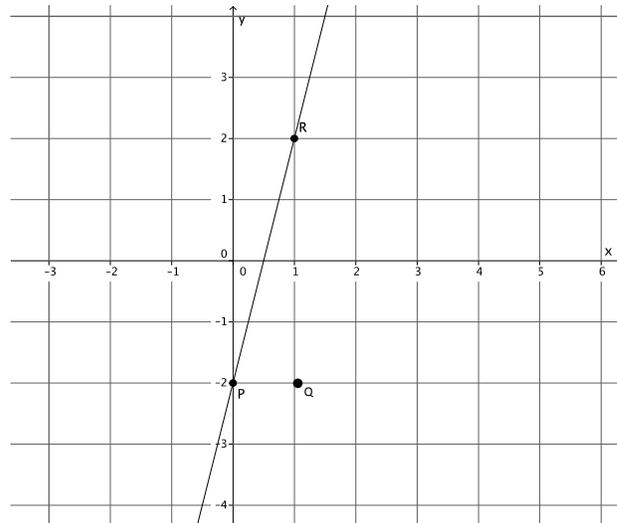


- The y -intercept point is $(0, -2)$.
- Select another point, R , on the line with integer coordinates.

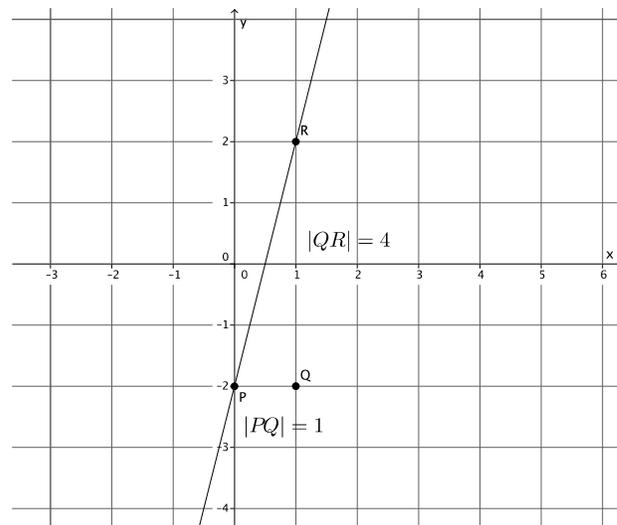


- Let R be the point located at $(1, 2)$.

- Now, place point Q , and find the slope of the line.



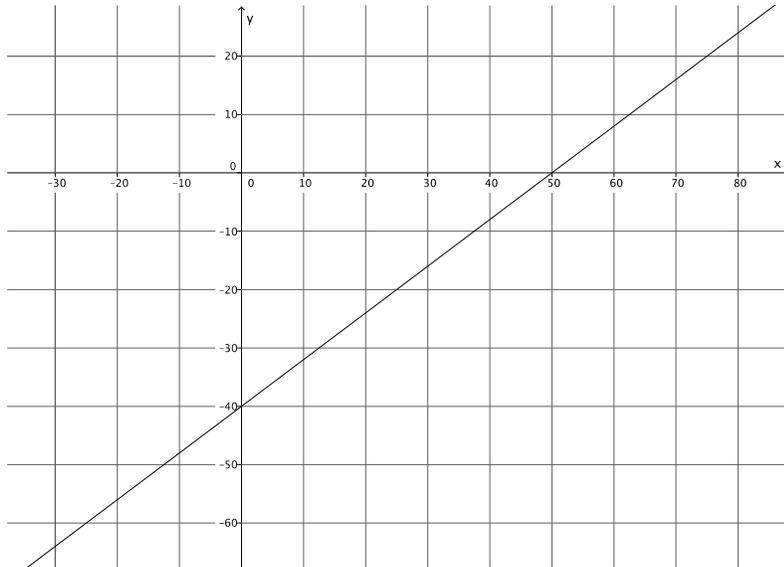
- The slope of the line is $m = 4$.
- Write the equation for the line.



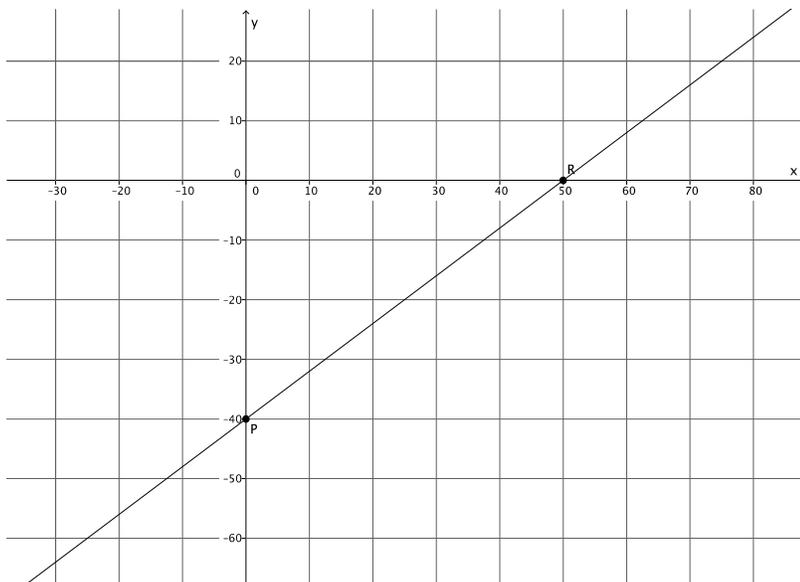
- The line is the graph of $y = 4x - 2$.

Example 3 (5 minutes)

- What is the y -intercept point of the line? Notice the units on the coordinate plane have increased.

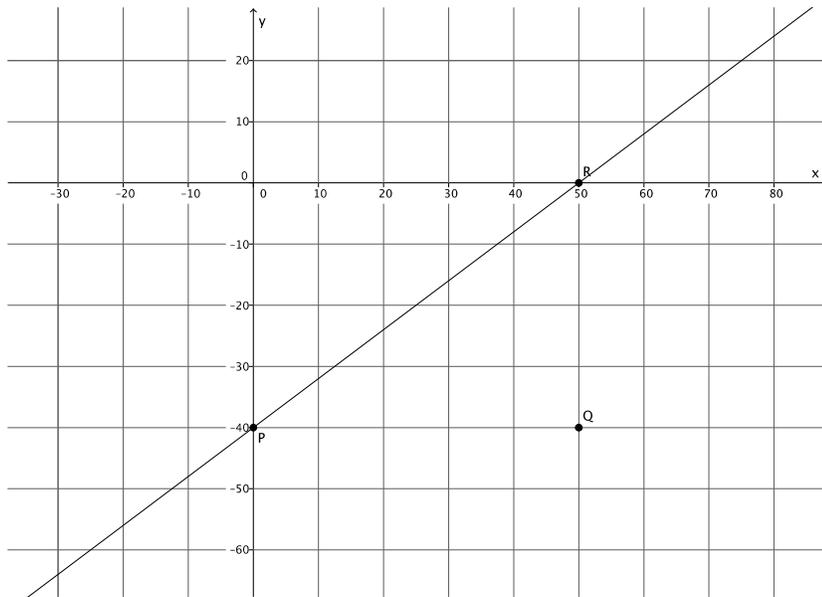


- The y -intercept point is $(0, -40)$.
- Select another point, R , on the line with integer coordinates.



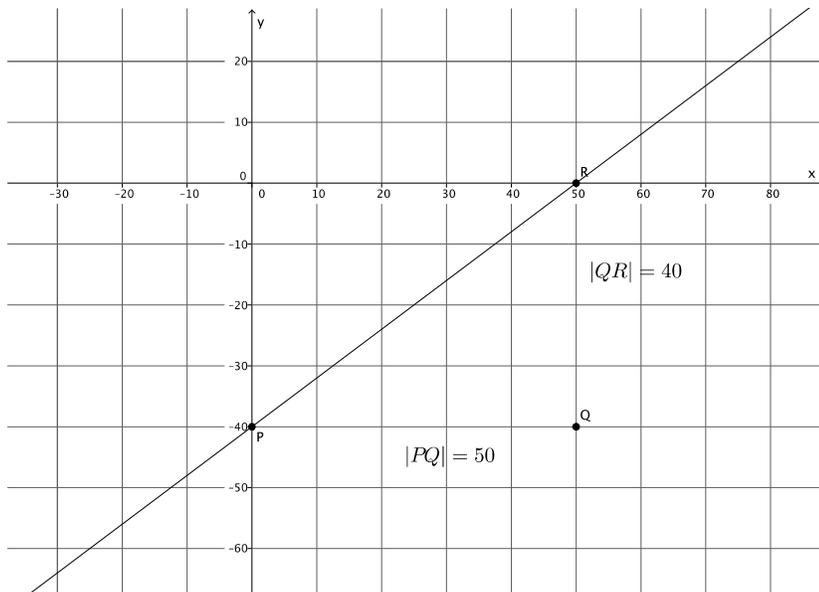
- Let R be $(50, 0)$.

- Now, place point Q , and find the slope of the line.



The slope of the line is $m = \frac{40}{50} = \frac{4}{5}$.

- Write the equation for the line.



The line is the graph of $y = \frac{4}{5}x - 40$.

- The last thing we will do to this linear equation is rewrite it in standard form $ax + by = c$, where a , b , and c are integers, and a is not negative. That means we must multiply the entire equation by a number that will turn $\frac{4}{5}$ into an integer. What number should we multiply by?

$$\square \quad \frac{4}{5}(5) = 4$$

- We multiply the entire equation by 5.

$$\begin{aligned} \left(y = \frac{4}{5}x - 40\right) 5 \\ 5y = 4x - 200 \\ -4x + 5y = 4x - 4x - 200 \\ -4x + 5y = -200 \\ -1(-4x + 5y = -200) \\ 4x - 5y = 200 \end{aligned}$$

The standard form of the linear equation is $4x - 5y = 200$.

Exercises (10 minutes)

Students complete Exercises 1–6 independently.

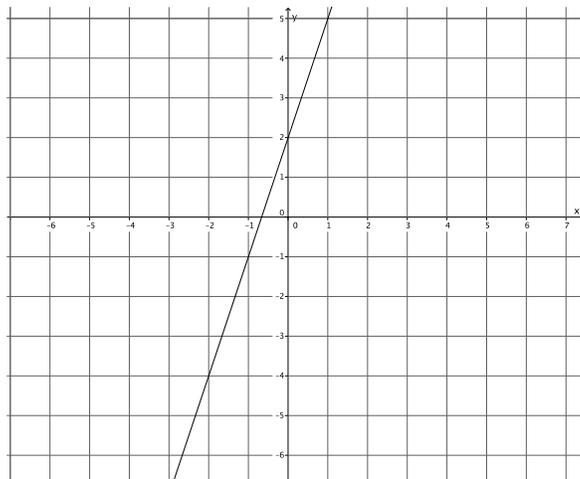
Exercises

- Write the equation that represents the line shown.

$$y = 3x + 2$$

Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers, and a is not negative.

$$\begin{aligned} y &= 3x + 2 \\ -3x + y &= 3x - 3x + 2 \\ -3x + y &= 2 \\ -1(-3x + y = 2) \\ 3x - y &= -2 \end{aligned}$$

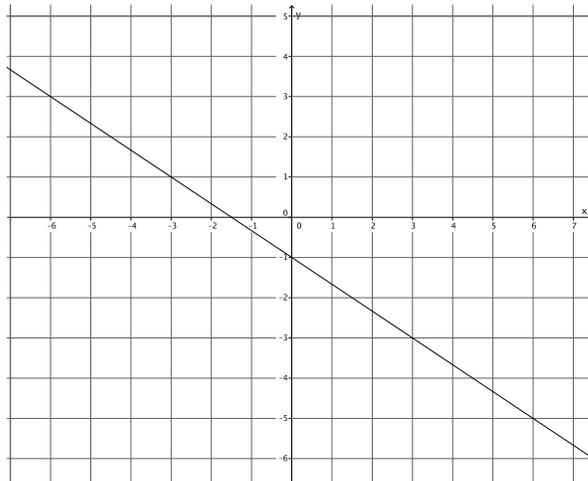


2. Write the equation that represents the line shown.

$$y = -\frac{2}{3}x - 1$$

Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers, and a is not negative.

$$\begin{aligned} y &= -\frac{2}{3}x - 1 \\ \left(y = -\frac{2}{3}x - 1\right) 3 \\ 3y &= -2x - 3 \\ 2x + 3y &= -2x + 2x - 3 \\ 2x + 3y &= -3 \end{aligned}$$

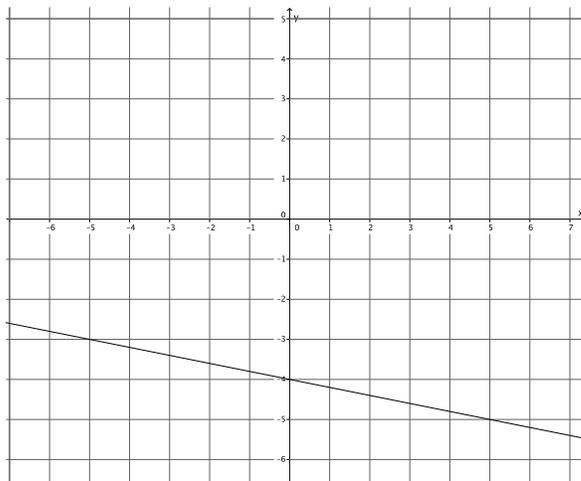


3. Write the equation that represents the line shown.

$$y = -\frac{1}{5}x - 4$$

Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers, and a is not negative.

$$\begin{aligned} y &= -\frac{1}{5}x - 4 \\ \left(y = -\frac{1}{5}x - 4\right) 5 \\ 5y &= -x - 20 \\ x + 5y &= -x + x - 20 \\ x + 5y &= -20 \\ x + 5 & \end{aligned}$$

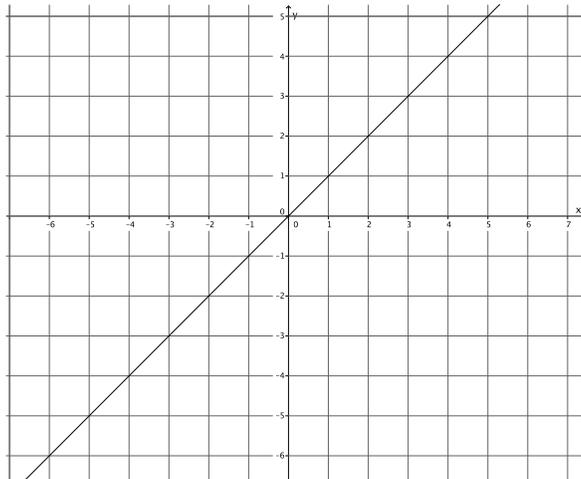


4. Write the equation that represents the line shown.

$$y = x$$

Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers, and a is not negative.

$$\begin{aligned} y &= x \\ -x + y &= x - x \\ -x + y &= 0 \\ -1(-x + y) &= 0 \\ x - y &= 0 \end{aligned}$$

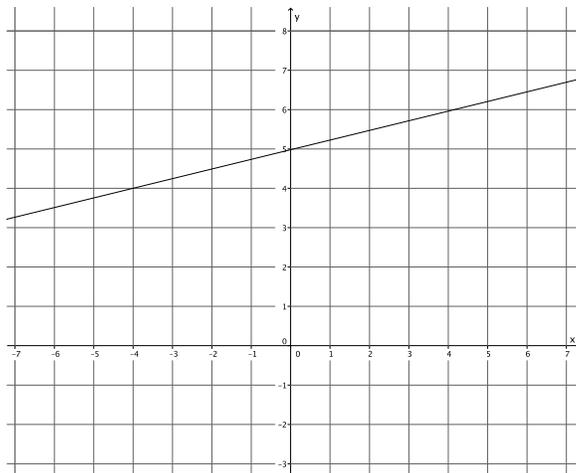


5. Write the equation that represents the line shown.

$$y = \frac{1}{4}x + 5$$

Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers, and a is not negative.

$$\begin{aligned} y &= \frac{1}{4}x + 5 \\ \left(y = \frac{1}{4}x + 5\right) 4 & \\ 4y &= x + 20 \\ -x + 4y &= x - x + 20 \\ -x + 4y &= 20 \\ -1(-x + 4y) &= 20 \\ x - 4y &= -20 \end{aligned}$$



6. Write the equation that represents the line shown.

$$y = -\frac{8}{5}x - 7$$

Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers, and a is not negative.

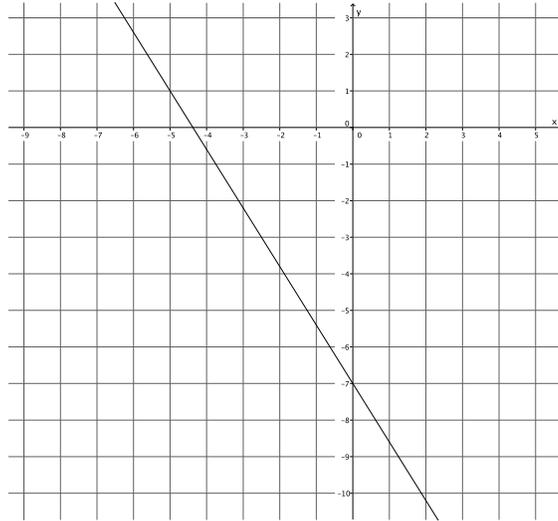
$$y = -\frac{8}{5}x - 7$$

$$\left(y = -\frac{8}{5}x - 7\right) 5$$

$$5y = -8x - 35$$

$$8x + 5y = -8x + 8x - 35$$

$$8x + 5y = -35$$



Closing (5 minutes)

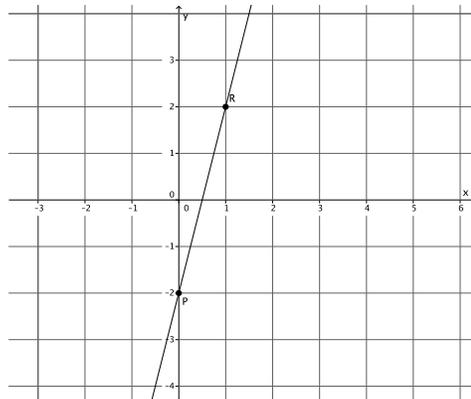
Summarize, or ask students to summarize, the main points from the lesson:

- We know that every line is a graph of a linear equation.
- We know how to use the y -intercept point and the slope of a line to write the equation of a line.

Lesson Summary

Write the equation of a line by determining the y -intercept point, $(0, b)$, and the slope, m , and replacing the numbers b and m into the equation $y = mx + b$.

Example:



The y -intercept point of this graph is $(0, -2)$.

The slope of this graph is $m = \frac{4}{1} = 4$.

The equation that represents the graph of this line is $y = 4x - 2$.

Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers, and a is not negative.

Exit Ticket (5 minutes)

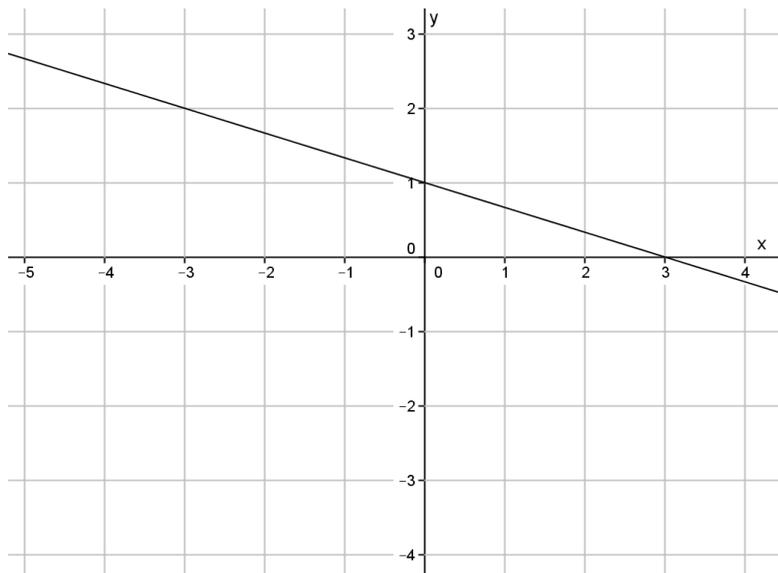
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Lesson 20: Every Line Is a Graph of a Linear Equation

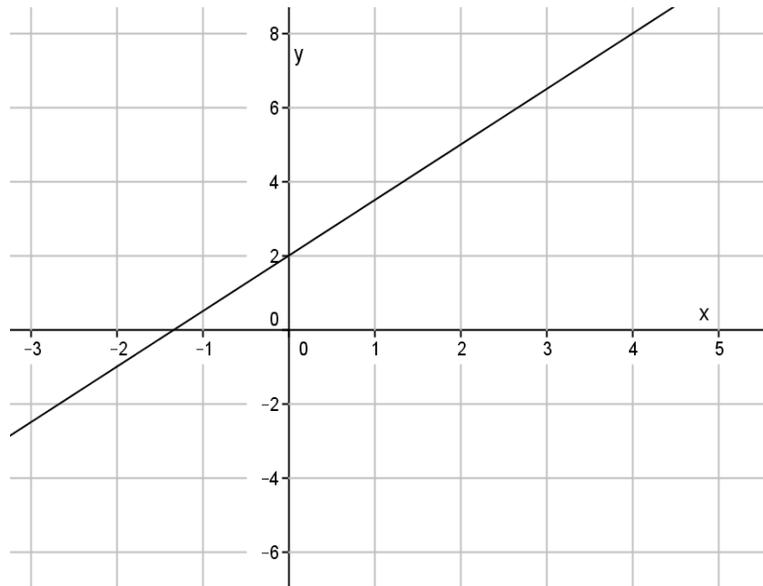
Exit Ticket

1. Write an equation in slope-intercept form that represents the line shown.



2. Use the properties of equality to change the equation you wrote for Problem 1 from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers, and a is not negative.

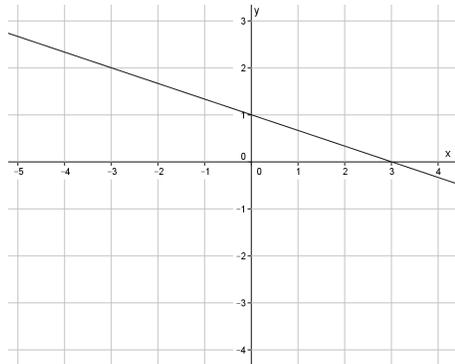
3. Write an equation in slope-intercept form that represents the line shown.



4. Use the properties of equality to change the equation you wrote for Problem 3 from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers, and a is not negative.

Exit Ticket Sample Solutions

1. Write an equation in slope-intercept form that represents the line shown.

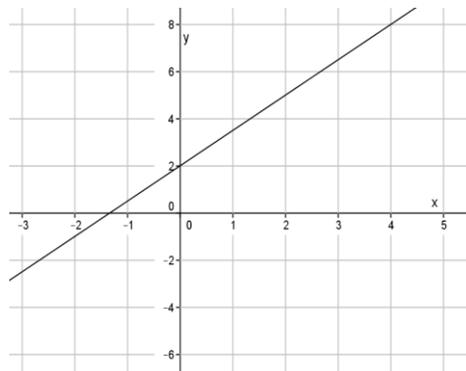


$$y = -\frac{1}{3}x + 1$$

2. Use the properties of equality to change the equation you wrote for Problem 1 from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers, and a is not negative.

$$\begin{aligned} y &= -\frac{1}{3}x + 1 \\ \left(y = -\frac{1}{3}x + 1\right) 3 \\ 3y &= -x + 3 \\ x + 3y &= -x + x + 3 \\ x + 3y &= 3 \end{aligned}$$

3. Write an equation in slope-intercept form that represents the line shown.



$$y = \frac{3}{2}x + 2$$

4. Use the properties of equality to change the equation you wrote for Problem 3 from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers, and a is not negative.

$$\begin{aligned}
 y &= \frac{3}{2}x + 2 \\
 \left(y = \frac{3}{2}x + 2\right) 2 \\
 2y &= 3x + 4 \\
 -3x + 2y &= 3x - 3x + 4 \\
 -3x + 2y &= 4 \\
 -1(-3x + 2y = 4) \\
 3x - 2y &= -4
 \end{aligned}$$

Problem Set Sample Solutions

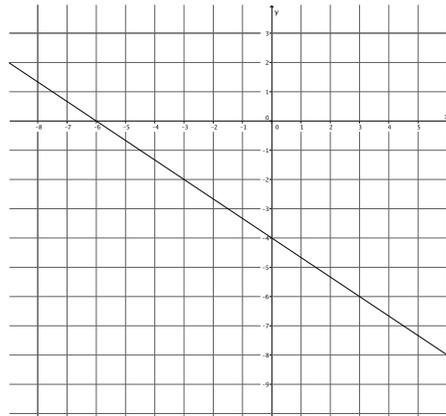
Students practice writing equations for lines.

1. Write the equation that represents the line shown.

$$y = -\frac{2}{3}x - 4$$

Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers, and a is not negative.

$$\begin{aligned}
 y &= -\frac{2}{3}x - 4 \\
 \left(y = -\frac{2}{3}x - 4\right) 3 \\
 3y &= -2x - 12 \\
 2x + 3y &= -2x + 2x - 12 \\
 2x + 3y &= -12
 \end{aligned}$$

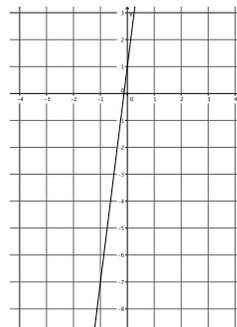


2. Write the equation that represents the line shown.

$$y = 8x + 1$$

Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers, and a is not negative.

$$\begin{aligned}
 y &= 8x + 1 \\
 -8x + y &= 8x - 8x + 1 \\
 -8x + y &= 1 \\
 -1(-8x + y = 1) \\
 8x - y &= -1
 \end{aligned}$$

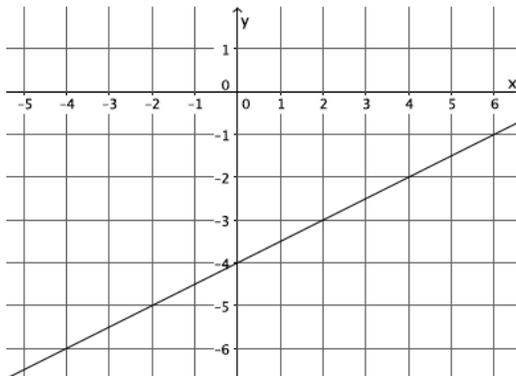


3. Write the equation that represents the line shown.

$$y = \frac{1}{2}x - 4$$

Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers, and a is not negative.

$$\begin{aligned} y &= \frac{1}{2}x - 4 \\ \left(y = \frac{1}{2}x - 4\right) 2 \\ 2y &= x - 8 \\ -x + 2y &= x - x - 8 \\ -x + 2y &= -8 \\ -1(-x + 2y &= -8) \\ x - 2y &= 8 \end{aligned}$$

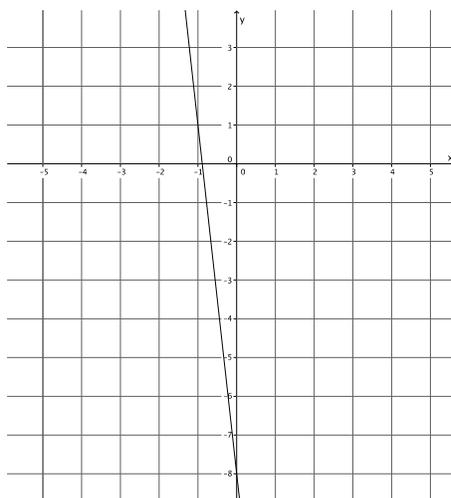


4. Write the equation that represents the line shown.

$$y = -9x - 8$$

Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers, and a is not negative.

$$\begin{aligned} y &= -9x - 8 \\ 9x + y &= -9x + 9x - 8 \\ 9x + y &= -8 \end{aligned}$$

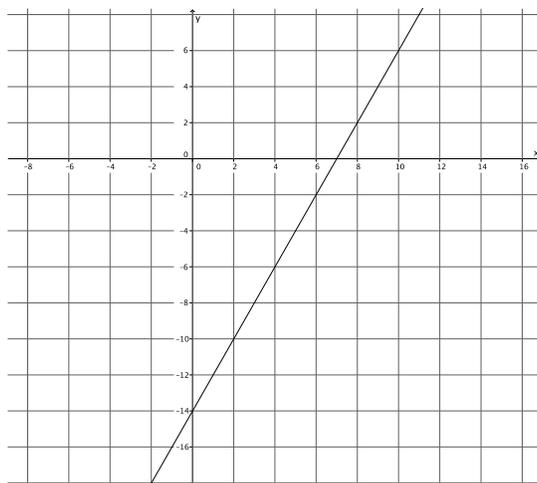


5. Write the equation that represents the line shown.

$$y = 2x - 14$$

Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers, and a is not negative.

$$\begin{aligned} y &= 2x - 14 \\ -2x + y &= 2x - 2x - 14 \\ -2x + y &= -14 \\ -1(-2x + y &= -14) \\ 2x - y &= 14 \end{aligned}$$



6. Write the equation that represents the line shown.

$$y = -5x + 45$$

Use the properties of equality to change the equation from slope-intercept form, $y = mx + b$, to standard form, $ax + by = c$, where a , b , and c are integers, and a is not negative.

$$y = -5x + 45$$

$$5x + y = -5x + 5x + 45$$

$$5x + y = 45$$

