Lesson 6: Solutions of a Linear Equation

Student Outcomes

- Students transform equations into simpler forms using the distributive property.
- Students learn that not every linear equation has a solution.

Lesson Notes

The distributive property can be used to both expand and simplify expressions. Students have already used the distributive property to "collect like terms." For example, $2x + 6x = (2 + 6)x = 8x$. Students have also used the distributive property to expand expressions. For example, $2(x + 5) = 2x + 10$. In this lesson, students continue to use the distributive property to solve more complicated equations. Also highlighted in this lesson is a common error that students make when using the distributive property, which is multiplying a factor to terms that are not part of the group. For example, in the expression $3(x + 1) - 5$, students should know that they do not distribute the factor 3 to the term -5 because it is not in the group $(x + 1)$.

Classwork

Example 1 (4 minutes)

- What value of x would make the linear equation $4x + 3(4x + 7) = 4(7x + 3) - 3$ true? What is the "best" first step, and why?

Have a discussion with students about what the "best" first step is and why. Make clear that the distributive property allows students to better see and work with the terms of the linear equation. Proceed with the following discussion.

- In order to find out what that solution might be, we must use the distributive property. The left side of the equation has the following expression:

$$4x + 3(4x + 7).$$

  Where and how will the distributive property be used?
  - *We will need to use the distributive property to expand* $3(4x + 7)$ *and then again to collect like terms.*
- Our work for now is just on the left side of the equation; the right side will remain unchanged for the moment.

$$4x + 3(4x + 7) = 4(7x + 3) - 3$$
$$4x + 12x + 21 = 4(7x + 3) - 3$$
$$(4 + 12)x + 21 = 4(7x + 3) - 3$$
$$16x + 21 = 4(7x + 3) - 3$$

- Now we need to rewrite the right side. Again, we will use the distributive property. The left side of the equation will remain unchanged.

$$16x + 21 = 4(7x + 3) - 3$$
$$16x + 21 = 28x + 12 - 3$$

- Notice that we did not apply the distributive property to the term -3. Since it was not part of the group $(7x + 3)$, it is not multiplied by 4.

$$16x + 21 = 28x + 9$$

- Now we have transformed the given equation into the following form: $16x + 21 = 28x + 9$. Solve the equation.
  - *Student work:*

$$16x + 21 = 28x + 9$$
$$16x - 16x + 21 = 28x - 16x + 9$$
$$21 = 12x + 9$$
$$21 - 9 = 12x + 9 - 9$$
$$12 = 12x$$
$$x = 1$$

- Is $x = 1$ really a solution to the equation $4x + 3(4x + 7) = 4(7x + 3) - 3$? How do you know?
  - *Yes, $x = 1$ is a solution because $4 + 3(11) = 37$ and $4(10) - 3 = 37$. Since both expressions are equal to 37, then $x = 1$ is a solution to the equation.*

Example 2 (4 minutes)

- What value of x would make the following linear equation true: $20 - (3x - 9) - 2 = -(-11x + 1)$? Since we have a group of terms that is preceded by a "$-$" sign, we will simplify this first. The "$-$" sign means we need to take the opposite of each of the terms within the group (i.e., parentheses).
- We begin with the left side of the equation:

$$20 - (3x - 9) - 2 = -(-11x + 1).$$

We need only to take the opposite of the terms within the grouping symbols. Is the term -2 within the grouping symbol?
  - *No*
- For that reason, we need only find the opposite of $3x - 9$. What is the opposite of $3x - 9$?
  - *The opposite of $3x - 9$ is $-3x + 9$.*
- The left side of the equation is rewritten as

$$20 - (3x - 9) - 2 = -(-11x + 1)$$
$$20 - 3x + 9 - 2 = -(-11x + 1)$$
$$20 + 9 - 2 - 3x = -(-11x + 1)$$
$$27 - 3x = -(-11x + 1).$$

> *Scaffolding:*
> The equation in this example can be modified to $20 - (3x - 9 + 1) = 10$ to meet the needs of diverse learners. Also, consider having students fold a piece of paper in half, solve on the left side, and justify their steps on the right side.

- Now we rewrite the right side of the equation: $-(-11x + 1)$.

$$27 - 3x = -(-11x + 1)$$
$$27 - 3x = 11x - 1$$

- The transformed equation is $27 - 3x = 11x - 1$.

$$27 - 3x = 11x - 1$$
$$27 - 3x + 3x = 11x + 3x - 1$$
$$27 = 14x - 1$$
$$27 + 1 = 14x - 1 + 1$$
$$28 = 14x$$
$$2 = x$$

- Check: The left side is $20 - (3x - 9) - 2 = 20 - (3(2) - 9) - 2 = 20 - (-3) - 2 = 20 + 3 - 2 = 21$. The right side is $-(-11x + 1) = -(-11(2) + 1) = -(-22 + 1) = -(-21) = 21$. Since $21 = 21$, $x = 2$ is the solution.

Example 3 (4 minutes)

- What value of x would make the following linear equation true: $\frac{1}{2}(4x + 6) - 2 = -(5x + 9)$? Begin by transforming both sides of the equation into a simpler form.
  - *Student work:*

$$\frac{1}{2}(4x + 6) - 2 = -(5x + 9)$$
$$2x + 3 - 2 = -5x - 9$$

Scaffolding:

The equation in this example can be modified to $2x + 1 = -(5x + 9)$ to meet the needs of diverse learners.

Make sure that students do not distribute the factor $\frac{1}{2}$ to the term -2 and that they have, in general, transformed the equation correctly.

- Now that we have the simpler equation, $2x + 3 - 2 = -5x - 9$, complete the solution.
  - *Student work:*

$$2x + 3 - 2 = -5x - 9$$
$$2x + 1 = -5x - 9$$
$$2x + 1 - 1 = -5x - 9 - 1$$
$$2x + 0 = -5x - 10$$
$$2x = -5x - 10$$
$$2x + 5x = -5x + 5x - 10$$
$$(2 + 5)x = (-5 + 5)x - 10$$
$$7x = 0x - 10$$
$$7x = -10$$
$$\frac{7}{7}x = -\frac{10}{7}$$
$$x = -\frac{10}{7}$$

- Check: The left side is
$$\frac{1}{2}(4x+6)-2=\frac{1}{2}\left(4\left(-\frac{10}{7}\right)+6\right)-2=\frac{1}{2}\left(-\frac{40}{7}+6\right)-2=\frac{1}{2}\left(\frac{2}{7}\right)-2=\frac{1}{7}-2=-\frac{13}{7}.$$
The right side is
$$-(5x+9)=-\left(5\left(-\frac{10}{7}\right)+9\right)=-\left(-\frac{50}{7}+9\right)=-\left(\frac{13}{7}\right)=-\frac{13}{7}.$$
Since $-\frac{13}{7}=-\frac{13}{7}$, $x=-\frac{10}{7}$ is the solution.

Example 4 (11 minutes)

- Consider the following equation: $2(x+1)=2x-3$. What value of x makes the equation true?
  - *Student work:*
$$2(x+1)=2x-3$$
$$2x+2=2x-3$$
$$2x-2x+2=2x-2x-3$$
$$2=-3$$

MP.3

- How should we interpret $2=-3$?

Lead a discussion with the conclusion that since $2\neq-3$, then the equation has no solution. Allow students time to try to find a value of x that would make it true by guessing and checking. After they realize that there is no such number x, make it clear to students that some equations have no solution. Ask the following question.

- Why do you think this happened?
  - *We know that an equation is a statement of equality. The linear expressions were such that they could not be equal to each other, no matter what value was substituted for x.*
- What value of x would make the following linear equation true: $9(4-2x)-3=4-6(3x-5)$? Transform the equation by simplifying both sides.
  - *Student work:*
$$9(4-2x)-3=4-6(3x-5)$$
$$36-18x-3=4-18x+30$$

Scaffolding:

The equation in this example can be modified to $9(4-2x)-3=-18x$ to meet the needs of diverse learners.

Be sure to check that students did not subtract $4-6$ on the right side and then distribute -2. This is a common error. Remind students that they must multiply first and then add or subtract, just like they would to simplify expressions using the correct order of operations.

- The transformed equation is $36 - 18x - 3 = 4 - 18x + 30$. Now, complete the solution.
  - *Student work:*

$$36 - 18x - 3 = 4 - 18x + 30$$
$$33 - 18x = 34 - 18x$$
$$33 - 18x + 18x = 34 - 18x + 18x$$
$$33 = 34$$

Like the last problem, there is no value of x that can be substituted into the equation to make it true. Therefore, this equation has no solution.

- Write at least one equation that has no solution. It does not need to be complicated, but the result should be similar to the last two problems. The result from the first equation was $2 \neq -3$, and the second was $33 \neq 34$.

Have students share their equations and verify that they have no solution.

Example 5 (Optional – 4 minutes)

- So far, we have used the distributive property to simplify expressions when solving equations. In some cases, we can use the distributive property to make our work even simpler. Consider the following equation:

$$3x + 15 = -6.$$

Notice that each term has a common factor of 3. We will use the distributive property and what we know about the properties of equality to solve this equation quickly.

$$3x + 15 = -6$$
$$3(x + 5) = 3 \cdot (-2)$$

Notice that the expressions on both sides of the equal sign have a factor of 3. We can use the multiplication property of equality, *if $A = B$, then $A \cdot C = B \cdot C$ as follows*:

$$3(x + 5) = 3 \cdot (-2)$$
$$x + 5 = -2$$
$$x + 5 - 5 = -2 - 5$$
$$x = -7.$$

- This is not something that we can expect to do every time we solve an equation, but it is good to keep an eye out for it.

Exercises (12 minutes)

Students complete Exercises 1–6 independently.

Exercises

Find the value of x that makes the equation true.

1. $17 - 5(2x - 9) = -(-6x + 10) + 4$

$$17 - 5(2x - 9) = -(-6x + 10) + 4$$
$$17 - 10x + 45 = 6x - 10 + 4$$
$$62 - 10x = 6x - 6$$
$$62 - 10x + 10x = 6x + 10x - 6$$
$$62 = 16x - 6$$
$$62 + 6 = 16x - 6 + 6$$
$$68 = 16x$$
$$\frac{68}{16} = \frac{16}{16}x$$
$$\frac{68}{16} = x$$
$$\frac{17}{4} = x$$

2. $-(x - 7) + \frac{5}{3} = 2(x + 9)$

$$-(x - 7) + \frac{5}{3} = 2(x + 9)$$
$$-x + 7 + \frac{5}{3} = 2x + 18$$
$$-x + \frac{26}{3} = 2x + 18$$
$$-x + x + \frac{26}{3} = 2x + x + 18$$
$$\frac{26}{3} = 3x + 18$$
$$\frac{26}{3} - 18 = 3x + 18 - 18$$
$$-\frac{28}{3} = 3x$$
$$\frac{1}{3} \cdot \frac{-28}{3} = \frac{1}{3} \cdot 3x$$
$$-\frac{28}{9} = x$$

3. $\frac{4}{9}+4(x-1)=\frac{28}{9}-(x-7x)+1$

$$\frac{4}{9}+4(x-1)=\frac{28}{9}-(x-7x)+1$$
$$\frac{4}{9}-\frac{4}{9}+4(x-1)=\frac{28}{9}-\frac{4}{9}-(x-7x)+1$$
$$4x-4=\frac{24}{9}-x+7x+1$$
$$4x-4=\frac{33}{9}+6x$$
$$4x-4+4=\frac{33}{9}+\frac{36}{9}+6x$$
$$4x=\frac{69}{9}+6x$$
$$4x-6x=\frac{69}{9}+6x-6x$$
$$-2x=\frac{23}{3}$$
$$\frac{1}{-2}\cdot -2x=\frac{1}{-2}\cdot\frac{23}{3}$$
$$x=-\frac{23}{6}$$

4. $5(3x+4)-2x=7x-3(-2x+11)$

$$5(3x+4)-2x=7x-3(-2x+11)$$
$$15x+20-2x=7x+6x-33$$
$$13x+20=13x-33$$
$$13x-13x+20=13x-13x-33$$
$$20\neq -33$$

This equation has no solution.

5. $7x-(3x+5)-8=\frac{1}{2}(8x+20)-7x+5$

$$7x-(3x+5)-8=\frac{1}{2}(8x+20)-7x+5$$
$$7x-3x-5-8=4x+10-7x+5$$
$$4x-13=-3x+15$$
$$4x-13+13=-3x+15+13$$
$$4x=-3x+28$$
$$4x+3x=-3x+3x+28$$
$$7x=28$$
$$x=4$$

6. Write at least three equations that have no solution.

Answers will vary. Verify that the equations written have no solution.

Closing (5 minutes)

Summarize, or ask students to summarize, the following main points from the lesson:

- We know how to transform equations into simpler forms using the distributive property.
- We now know that there are some equations that do not have solutions.

> **Lesson Summary**
>
> **The distributive property is used to expand expressions. For example, the expression $2(3x - 10)$ is rewritten as $6x - 20$ after the distributive property is applied.**
>
> **The distributive property is used to simplify expressions. For example, the expression $7x + 11x$ is rewritten as $(7 + 11)x$ and $18x$ after the distributive property is applied.**
>
> **The distributive property is applied only to terms within a group:**
>
> $$4(3x + 5) - 2 = 12x + 20 - 2.$$
>
> **Notice that the term -2 is not part of the group and, therefore, not multiplied by 4.**
>
> **When an equation is transformed into an untrue sentence, such as $5 \neq 11$, we say the equation has *no solution*.**

Exit Ticket (5 minutes)

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Lesson 6: Solutions of a Linear Equation

Exit Ticket

Transform the equation if necessary, and then solve to find the value of x that makes the equation true.

1. $5x - (x + 3) = \frac{1}{3}(9x + 18) - 5$

2. $5(3x + 9) - 2x = 15x - 2(x - 5)$

Exit Ticket Sample Solutions

Transform the equation if necessary, and then solve to find the value of x that makes the equation true.

1. $5x-(x+3)=\frac{1}{3}(9x+18)-5$

$$5x-(x+3)=\frac{1}{3}(9x+18)-5$$
$$5x-x-3=3x+6-5$$
$$4x-3=3x+1$$
$$4x-3x-3=3x-3x+1$$
$$x-3=1$$
$$x-3+3=1+3$$
$$x=4$$

2. $5(3x+9)-2x=15x-2(x-5)$

$$5(3x+9)-2x=15x-2(x-5)$$
$$15x+45-2x=15x-2x+10$$
$$13x+45=13x+10$$
$$13x-13x+45=13x-13x+10$$
$$45\neq 10$$

Since $45\neq 10$, the equation has no solution.

Problem Set Sample Solutions

Students practice using the distributive property to transform equations and solve.

Transform the equation if necessary, and then solve it to find the value of x that makes the equation true.

1. $x-(9x-10)+11=12x+3\left(-2x+\frac{1}{3}\right)$

$$x-(9x-10)+11=12x+3\left(-2x+\frac{1}{3}\right)$$
$$x-9x+10+11=12x-6x+1$$
$$-8x+21=6x+1$$
$$-8x+8x+21=6x+8x+1$$
$$21=14x+1$$
$$21-1=14x+1-1$$
$$20=14x$$
$$\frac{20}{14}=\frac{14}{14}x$$
$$\frac{10}{7}=x$$

2. $7x + 8\left(x + \frac{1}{4}\right) = 3(6x - 9) - 8$

$$7x + 8\left(x + \frac{1}{4}\right) = 3(6x - 9) - 8$$
$$7x + 8x + 2 = 18x - 27 - 8$$
$$15x + 2 = 18x - 35$$
$$15x - 15x + 2 = 18x - 15x - 35$$
$$2 = 3x - 35$$
$$2 + 35 = 3x - 35 + 35$$
$$37 = 3x$$
$$\frac{37}{3} = \frac{3}{3}x$$
$$\frac{37}{3} = x$$

3. $-4x - 2(8x + 1) = -(-2x - 10)$

$$-4x - 2(8x + 1) = -(-2x - 10)$$
$$-4x - 16x - 2 = 2x + 10$$
$$-20x - 2 = 2x + 10$$
$$-20x + 20x - 2 = 2x + 20x + 10$$
$$-2 = 22x + 10$$
$$-2 - 10 = 22x + 10 - 10$$
$$-12 = 22x$$
$$-\frac{12}{22} = \frac{22}{22}x$$
$$-\frac{6}{11} = x$$

4. $11(x + 10) = 132$

$$11(x + 10) = 132$$
$$\left(\frac{1}{11}\right)11(x + 10) = \left(\frac{1}{11}\right)132$$
$$x + 10 = 12$$
$$x + 10 - 10 = 12 - 10$$
$$x = 2$$

5. $37x + \frac{1}{2} - \left(x + \frac{1}{4}\right) = 9(4x - 7) + 5$

$$37x + \frac{1}{2} - \left(x + \frac{1}{4}\right) = 9(4x - 7) + 5$$
$$37x + \frac{1}{2} - x - \frac{1}{4} = 36x - 63 + 5$$
$$36x + \frac{1}{4} = 36x - 58$$
$$36x - 36x + \frac{1}{4} = 36x - 36x - 58$$
$$\frac{1}{4} \neq -58$$

This equation has no solution.

6. $3(2x - 14) + x = 15 - (-9x - 5)$

$$3(2x - 14) + x = 15 - (-9x - 5)$$
$$6x - 42 + x = 15 + 9x + 5$$
$$7x - 42 = 20 + 9x$$
$$7x - 7x - 42 = 20 + 9x - 7x$$
$$-42 = 20 + 2x$$
$$-42 - 20 = 20 - 20 + 2x$$
$$-62 = 2x$$
$$-31 = x$$

7. $8(2x + 9) = 56$

$$8(2x + 9) = 56$$
$$\left(\frac{1}{8}\right)8(2x + 9) = \left(\frac{1}{8}\right)56$$
$$2x + 9 = 7$$
$$2x + 9 - 9 = 7 - 9$$
$$2x = -2$$
$$\left(\frac{1}{2}\right)2x = \left(\frac{1}{2}\right) - 2$$
$$x = -1$$