Lesson 3: Linear Equations in x

Student Outcomes

- Students know that a linear equation is a statement of equality between two expressions.
- Students know that a linear equation in x is actually a question: Can you find all numbers x, if they exist, that satisfy a given equation? Students know that those numbers x that satisfy a given equation are called *solutions*.

Classwork

Concept Development (7 minutes)

- We want to define "linear equation in x." Here are some examples of linear equations in x. Using what you know about the words *linear* (from Lesson 2) and *equation* (from Lesson 1), develop a mathematical definition of "linear equation in x."

> *Scaffolding:*
> Consider developing a word bank or word wall to be used throughout the module.

Show students the examples below, and provide them time to work individually or in small groups to develop an appropriate definition. Once students share their definitions, continue with the definition and discussion that follows.

$x + 11 = 15$	$5 + 3 = 8$	$-\frac{1}{2}x = 22$
$15 - 4x = x + \frac{4}{5}$	$3 - (x + 2) = -12x$	$\frac{3}{4}x + 6(x - 1) = 9(2 - x)$

- When two linear expressions are equal, they can be written as a linear equation in x.
- Consider the following equations. Which are true, and how do you know?

$$4 + 1 = 5$$
$$6 + 5 = 16$$
$$21 - 6 = 15$$
$$6 - 2 = 1$$

  - *The first and third equations are true because the value on the left side is equal to the number on the right side.*
- Is $4 + 15x = 49$ true? How do you know?

Have a discussion that leads to students developing a list of values for x that make it false, along with one value of x that makes it true. Then, conclude the discussion by making the two points below.

- A linear equation in x is a statement about equality, but it is also an invitation to find all of the numbers x, if they exist, that make the equation true. Sometimes the question is asked in this way: What number(s) x satisfy the equation? The question is often stated more as a directive: Solve. When phrased as a directive, it is still considered a question. Is there a number(s) x that make the statement true? If so, what is the number(s) x?

- Equations that contain a variable do not have a definitive truth value; in other words, there are values of the variable that make the equation a true statement and values of the variable that make it a false statement. When we say that we have "solved an equation," what we are really saying is that we have found a number (or numbers) x that make the equation true. That number x is called the *solution* to the equation.

Example 1 (4 minutes)

- Here is a linear equation in x: $4 + 15x = 49$. Is there a number x that makes the linear expression $4 + 15x$ equal to the linear expression 49? Suppose you are told this number x has a value of 2, that is, $x = 2$. We replace any instance of x in the linear equation with the value of 2, as shown:

$$4 + 15 \cdot 2 = 49.$$

Next, we evaluate each side of the equation. The left side is

$$\begin{aligned} 4 + 15 \cdot 2 &= 4 + 30 \\ &= 34. \end{aligned}$$

The right side of the equation is 49. Clearly, $34 \neq 49$. Therefore, the number 2 is not a solution to this equation.

- Is the number 3 a solution to the equation? That is, is this equation a true statement when $x = 3$?
  - *Yes, because the left side of the equation equals the right side of the equation when $x = 3$.*

    The left side is

$$\begin{aligned} 4 + 15 \cdot 3 &= 4 + 45 \\ &= 49. \end{aligned}$$

The right side is 49. Since $49 = 49$, then we can say that $x = 3$ is a solution to the equation $4 + 15x = 49$.

- 3 is a solution to the equation because it is a value of x that makes the equation a true statement.

> *Scaffolding:*
>
> Remind students that when a number and a symbol are next to one another, such as $15x$, it is not necessary to use a symbol to represent the multiplication (it is a convention). For clarity, when two numbers are being multiplied, it is necessary to use a multiplication symbol. For example, it is necessary to tell the difference between the number, 152, and the product, $15 \cdot 2$.

Example 2 (4 minutes)

- Here is a linear equation in x: $8x - 19 = -4 - 7x$.
- Is 5 a solution to the equation? That is, is the equation a true statement when $x = 5$?
  - *No, because the left side of the equation does not equal the right side of the equation when $x = 5$.*

    The left side is

$$\begin{aligned} 8 \cdot 5 - 19 &= 40 - 19 \\ &= 21. \end{aligned}$$

The right side is

$$\begin{aligned} -4 - 7 \cdot 5 &= -4 - 35 \\ &= -39. \end{aligned}$$

Since $21 \neq -39$, then $x \neq 5$. That is, 5 is not a solution to the equation.

- Is 1 a solution to the equation? That is, is this equation a true statement when $x = 1$?
  - *Yes. The left side and right side of the equation are equal to the same number when $x = 1$.*

    The left side is

    $$\begin{aligned} 8 \cdot 1 - 19 &= 8 - 19 \\ &= -11. \end{aligned}$$

    The right side is

    $$\begin{aligned} -4 - 7 \cdot 1 &= -4 - 7 \\ &= -11. \end{aligned}$$

    Since $-11 = -11$, then $x = 1$. That is, 1 is a solution to the equation.

Example 3 (4 minutes)

- Here is a linear equation in x: $3(x + 9) = 4x - 7 + 7x$.
- We can make our work simpler if we use some properties to transform the expression on the right side of the equation into an expression with fewer terms.

Provide students time to transform the equation into fewer terms, and then proceed with the points below.

MP.1

- For example, notice that on the right side, there are two terms that contain x. First, we will use the commutative property to rearrange the terms to better see what we are doing.

  $$4x + 7x - 7$$

  Next, we will use the distributive property to collect the terms that contain x.

  $$\begin{aligned} 4x + 7x - 7 &= (4 + 7)x - 7 \\ &= 11x - 7 \end{aligned}$$

  Finally, the transformed (but still the same) equation can be written as $3(x + 9) = 11x - 7$.

- Is $\frac{5}{4}$ a solution to the equation? That is, is this equation a true statement when $x = \frac{5}{4}$?
  - *No, because the left side of the equation does not equal the right side of the equation when $x = \frac{5}{4}$.*

    The left side is

    $$\begin{aligned} 3\left(\frac{5}{4} + 9\right) &= 3\left(\frac{41}{4}\right) \\ &= \frac{123}{4}. \end{aligned}$$

    The right side is

    $$\begin{aligned} 11 \cdot \frac{5}{4} - 7 &= \frac{55}{4} - 7 \\ &= \frac{27}{4}. \end{aligned}$$

    Since $\frac{123}{4} \neq \frac{27}{4}$, then $x \neq \frac{5}{4}$. That is, $\frac{5}{4}$ is not a solution to the equation.

Example 4 (4 minutes)

- Here is a linear equation in x: $-2x + 11 - 5x = 5 - 6x$.
- We want to check to see if 6 is a solution to the equation; that is, is this equation a true statement when $x = 6$? Before we do that, what would make our work easier?
  - *We could use the commutative and distributive properties to transform the left side of the equation into an expression with fewer terms.*

$$\begin{aligned} -2x + 11 - 5x &= -2x - 5x + 11 \\ &= (-2 - 5)x + 11 \\ &= -7x + 11 \end{aligned}$$

- The transformed equation can be written as $-7x + 11 = 5 - 6x$. Is 6 a solution to the equation; that is, is this equation a true statement when $x = 6$?
  - *Yes, because the left side of the equation is equal to the right side of the equation when $x = 6$.*

    The left side is

$$\begin{aligned} -7x + 11 &= -7 \cdot 6 + 11 \\ &= -42 + 11 \\ &= -31. \end{aligned}$$

The right side is

$$\begin{aligned} 5 - 6x &= 5 - 6 \cdot 6 \\ &= 5 - 36 \\ &= -31. \end{aligned}$$

Since $-31 = -31$, then $x = 6$. That is, 6 is a solution to the equation.

Exercises (12 minutes)

Students complete Exercises 1–7 independently.

Exercises

1. **Is the equation a true statement when $x = -3$? In other words, is -3 a solution to the equation $6x + 5 = 5x + 8 + 2x$? Explain.**

   If we replace x with the number -3, then the left side of the equation is

$$\begin{aligned} 6 \cdot (-3) + 5 &= -18 + 5 \\ &= -13, \end{aligned}$$

and the right side of the equation is

$$\begin{aligned} 5 \cdot (-3) + 8 + 2 \cdot (-3) &= -15 + 8 - 6 \\ &= -7 - 6 \\ &= -13. \end{aligned}$$

Since $-13 = -13$, then $x = -3$ is a solution to the equation $6x + 5 = 5x + 8 + 2x$.

Note: Some students may have transformed the equation.

2. **Does $x = 12$ satisfy the equation $16 - \frac{1}{2}x = \frac{3}{4}x + 1$? Explain.**

If we replace x with the number 12, then the left side of the equation is

$$\begin{aligned} 16 - \frac{1}{2}x &= 16 - \frac{1}{2} \cdot (12) \\ &= 16 - 6 \\ &= 10, \end{aligned}$$

and the right side of the equation is

$$\begin{aligned} \frac{3}{4}x + 1 &= \frac{3}{4} \cdot (12) + 1 \\ &= 9 + 1 \\ &= 10. \end{aligned}$$

Since $10 = 10$, then $x = 12$ is a solution to the equation $16 - \frac{1}{2}x = \frac{3}{4}x + 1$.

3. **Chad solved the equation $24x + 4 + 2x = 3(10x - 1)$ and is claiming that $x = 2$ makes the equation true. Is Chad correct? Explain.**

If we replace x with the number 2, then the left side of the equation is

$$\begin{aligned} 24x + 4 + 2x &= 24 \cdot 2 + 4 + 2 \cdot 2 \\ &= 48 + 4 + 4 \\ &= 56, \end{aligned}$$

and the right side of the equation is

$$\begin{aligned} 3(10x - 1) &= 3(10 \cdot 2 - 1) \\ &= 3(20 - 1) \\ &= 3(19) \\ &= 57. \end{aligned}$$

Since $56 \neq 57$, then $x = 2$ is not a solution to the equation $24x + 4 + 2x = 3(10x - 1)$, and Chad is not correct.

4. **Lisa solved the equation $x + 6 = 8 + 7x$ and claimed that the solution is $x = -\frac{1}{3}$. Is she correct? Explain.**

If we replace x with the number $-\frac{1}{3}$, then the left side of the equation is

$$\begin{aligned} x + 6 &= -\frac{1}{3} + 6 \\ &= 5\frac{2}{3}, \end{aligned}$$

and the right side of the equation is

$$\begin{aligned} 8 + 7x &= 8 + 7 \cdot \left(-\frac{1}{3}\right) \\ &= 8 - \frac{7}{3} \\ &= \frac{24}{3} - \frac{7}{3} \\ &= \frac{17}{3}. \end{aligned}$$

Since $5\frac{2}{3} = \frac{17}{3}$, then $x = -\frac{1}{3}$ is a solution to the equation $x + 6 = 8 + 7x$, and Lisa is correct.

5. **Angel transformed the following equation from $6x + 4 - x = 2(x + 1)$ to $10 = 2(x + 1)$. He then stated that the solution to the equation is $x = 4$. Is he correct? Explain.**

No, Angel is not correct. He did not transform the equation correctly. The expression on the left side of the equation $6x + 4 - x = 2(x + 1)$ would transform to

$$\begin{aligned} 6x + 4 - x &= 6x - x + 4 \\ &= (6 - 1)x + 4 \\ &= 5x + 4. \end{aligned}$$

If we replace x with the number 4, then the left side of the equation is

$$\begin{aligned} 5x + 4 &= 5 \cdot 4 + 4 \\ &= 20 + 4 \\ &= 24, \end{aligned}$$

and the right side of the equation is

$$\begin{aligned} 2(x + 1) &= 2(4 + 1) \\ &= 2(5) \\ &= 10. \end{aligned}$$

Since $24 \neq 10$, then $x = 4$ is not a solution to the equation $6x + 4 - x = 2(x + 1)$, and Angel is not correct.

6. **Claire was able to verify that $x = 3$ was a solution to her teacher's linear equation, but the equation got erased from the board. What might the equation have been? Identify as many equations as you can with a solution of $x = 3$.**

Answers will vary. Ask students to share their equations and justifications as to how they knew $x = 3$ would make a true number sentence.

7. **Does an equation always have a solution? Could you come up with an equation that does not have a solution?**

Answers will vary. Expect students to write equations that are false. Ask students to share their equations and justifications as to how they knew the equation they wrote did not have a solution. The concept of "no solution" is introduced in Lesson 6 and solidified in Lesson 7.

Closing (5 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know that equations are statements about equality. That is, the expression on the left side of the equal sign is equal to the expression on the right side of the equal sign.
- We know that a solution to a linear equation in x will be a number and that when all instances of x are replaced with the number, the left side will equal the right side.

Lesson Summary

An equation is a statement about equality between two expressions. If the expression on the left side of the equal sign has the same value as the expression on the right side of the equal sign, then you have a true equation.

A solution of a linear equation in x is a number, such that when all instances of x are replaced with the number, the left side will equal the right side. For example, 2 is a solution to $3x + 4 = x + 8$ because when $x = 2$, the left side of the equation is

$$\begin{aligned} 3x + 4 &= 3(2) + 4 \\ &= 6 + 4 \\ &= 10, \end{aligned}$$

and the right side of the equation is

$$\begin{aligned} x + 8 &= 2 + 8 \\ &= 10. \end{aligned}$$

Since $10 = 10$, then $x = 2$ is a solution to the linear equation $3x + 4 = x + 8$.

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 3: Linear Equations in x

Exit Ticket

1. Is 8 a solution to $\frac{1}{2}x + 9 = 13$? Explain.

2. Write three different equations that have $x = 5$ as a solution.

3. Is -3 a solution to the equation $3x - 5 = 4 + 2x$? Explain.

Exit Ticket Sample Solutions

1. Is 8 a solution to $\frac{1}{2}x + 9 = 13$? Explain.

   If we replace x with the number 8, then the left side is $\frac{1}{2}(8) + 9 = 4 + 9 = 13$, and the right side is 13. Since $13 = 13$, then $x = 8$ is a solution.

2. Write three different equations that have $x = 5$ as a solution.

   Answers will vary. Accept equations where $x = 5$ makes a true number sentence.

3. Is -3 a solution to the equation $3x - 5 = 4 + 2x$? Explain.

   If we replace x with the number -3, then the left side is $3(-3) - 5 = -9 - 5 = -14$. The right side is $4 + 2(-3) = 4 - 6 = -2$. Since $-14 \neq -2$, then -3 is not a solution of the equation.

Problem Set Sample Solutions

Students practice determining whether or not a given number is a solution to the linear equation.

1. Given that $2x + 7 = 27$ and $3x + 1 = 28$, does $2x + 7 = 3x + 1$? Explain.

   No, because a linear equation is a statement about equality. We are given that $2x + 7 = 27$, but $3x + 1 = 28$. Since each linear expression is equal to a different number, $2x + 7 \neq 3x + 1$.

2. Is -5 a solution to the equation $6x + 5 = 5x + 8 + 2x$? Explain.

   If we replace x with the number -5, then the left side of the equation is

   $$\begin{aligned} 6 \cdot (-5) + 5 &= -30 + 5 \\ &= -25, \end{aligned}$$

   and the right side of the equation is

   $$\begin{aligned} 5 \cdot (-5) + 8 + 2 \cdot (-5) &= -25 + 8 - 10 \\ &= -17 - 10 \\ &= -27. \end{aligned}$$

   Since $-25 \neq -27$, then -5 is not a solution of the equation $6x + 5 = 5x + 8 + 2x$.

   Note: Some students may have transformed the equation.

3. Does $x = 1.6$ satisfy the equation $6 - 4x = -\frac{x}{4}$? Explain.

   If we replace x with the number 1.6, then the left side of the equation is

   $$\begin{aligned} 6 - 4 \cdot 1.6 &= 6 - 6.4 \\ &= -0.4, \end{aligned}$$

   and the right side of the equation is

   $$-\frac{1.6}{4} = -0.4.$$

   Since $-0.4 = -0.4$, then $x = 1.6$ is a solution of the equation $6 - 4x = -\frac{x}{4}$.

4. Use the linear equation $3(x+1) = 3x+3$ to answer parts (a)–(d).

a. Does $x = 5$ satisfy the equation above? Explain.

If we replace x with the number 5, then the left side of the equation is

$$\begin{aligned} 3(5+1) &= 3(6) \\ &= 18, \end{aligned}$$

and the right side of the equation is

$$\begin{aligned} 3x+3 &= 3 \cdot 5 + 3 \\ &= 15+3 \\ &= 18. \end{aligned}$$

Since $18 = 18$, then $x = 5$ is a solution of the equation $3(x+1) = 3x+3$.

b. Is $x = -8$ a solution of the equation above? Explain.

If we replace x with the number -8, then the left side of the equation is

$$\begin{aligned} 3(-8+1) &= 3(-7) \\ &= -21, \end{aligned}$$

and the right side of the equation is

$$\begin{aligned} 3x+3 &= 3 \cdot (-8) + 3 \\ &= -24+3 \\ &= -21. \end{aligned}$$

Since $-21 = -21$, then $x = -8$ is a solution of the equation $3(x+1) = 3x+3$.

c. Is $x = \frac{1}{2}$ a solution of the equation above? Explain.

If we replace x with the number $\frac{1}{2}$, then the left side of the equation is

$$\begin{aligned} 3\left(\frac{1}{2}+1\right) &= 3\left(\frac{1}{2}+\frac{2}{2}\right) \\ &= 3\left(\frac{3}{2}\right) \\ &= \frac{9}{2}, \end{aligned}$$

and the right side of the equation is

$$\begin{aligned} 3x+3 &= 3 \cdot \left(\frac{1}{2}\right) + 3 \\ &= \frac{3}{2} + 3 \\ &= \frac{3}{2} + \frac{6}{2} \\ &= \frac{9}{2}. \end{aligned}$$

Since $\frac{9}{2} = \frac{9}{2}$, then $x = \frac{1}{2}$ is a solution of the equation $3(x+1) = 3x+3$.

d. What interesting fact about the equation $3(x+1) = 3x+3$ is illuminated by the answers to parts (a), (b), and (c)? Why do you think this is true?

Note to teacher: Ideally, students will notice that the equation $3(x+1) = 3x+3$ is an identity under the distributive law. The purpose of this problem is to prepare students for the idea that linear equations can have more than one solution, which is a topic of Lesson 7.