



## Lesson 1: Writing Equations Using Symbols

### Student Outcomes

- Students write mathematical statements using symbols to represent numbers.
- Students know that written statements can be written as more than one correct mathematical sentence.

### Lesson Notes

The content of this lesson continues to develop the skills and concepts presented in Grades 6 and 7. Specifically, this lesson builds on both **6.EE.B.7** (Solve real-world and mathematical problems by writing and solving equations of the form  $x + p = q$  and  $px = q$ ) and **7.EE.B.4** (Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ ).

### Classwork

#### Discussion (4 minutes)

Show students the text of a mathematical statement compared to the equation.

The number 1,157 is the sum of the squares of two consecutive odd integers divided by the difference between the two consecutive odd integers.

$$1157 = \frac{x^2 + (x + 2)^2}{(x + 2) - x}$$

MP.2

Ask students to write or share aloud (a) how these two are related, (b) which representation they prefer, and (c) why. Then, continue with the discussion that follows.

- Using letters to represent numbers in mathematical statements was introduced by René Descartes in the 1600s. In that era, people used only words to describe mathematical statements. The use of letters, or symbols, to represent numbers not only brought clarity to mathematical statements, it also expanded the horizons of mathematics.
- The reason we want to learn how to write a mathematical statement using symbols is to save time and labor. Imagine having to write the sentence: “The number 1,157 is the sum of the squares of two consecutive odd integers divided by the difference between the two consecutive odd integers.” Then, imagine having to write the subsequent sentences necessary to solve it; compare that to the following:

Let  $x$  represent the first odd integer. Then,

$$1157 = \frac{x^2 + (x + 2)^2}{(x + 2) - x}$$

- Notice that  $x$  is just a number. That means the square of  $x$  is also a number, along with the square of the next odd integer and the difference between the numbers. This is a symbolic statement about numbers.
- Writing in symbols is simpler than writing in words, as long as everyone involved is clear about what the symbols mean. This lesson focuses on accurately transcribing written statements into mathematical symbols. When we write mathematical statements using letters, we say we are using symbolic language.

- All of the mathematical statements in this lesson are equations. Recall that an equation is a statement of equality between two expressions. Developing equations from written statements forms an important basis for problem solving and is one of the most vital parts of algebra. Throughout this module, there will be work with written statements and symbolic language. We will work first with simple expressions, then with equations that gradually increase in complexity, and finally with systems of equations (more than one equation at a time).

### Example 1 (3 minutes)

Throughout Example 1, have students write their thoughts on personal white boards or a similar tool and show their transcription(s).

- We want to express the following statement using symbolic language: A whole number has the property that when the square of half the number is subtracted from five times the number, we get the number itself.
- Do the first step, and hold up your personal white board.
  - First, we define the variable. Let  $x$  be the whole number.
- Using  $x$  to represent the whole number, write “the square of half the number.”
  - $\left(\frac{x}{2}\right)^2$  or  $\left(\frac{1}{2}x\right)^2$  or  $\left(\frac{x^2}{4}\right)$  or  $\left(\frac{1}{4}x^2\right)$

#### Scaffolding:

Alternative statement:

- A whole number has the property that when half the number is added to 15, we get the number itself.
- $\frac{1}{2}x + 15 = x$

Ask students to write their expressions in more than one way. Then, have students share their expressions of “the square of half the number.” Elicit the above responses from students (or provide them). Ask students why they are all correct.

- Write the entire statement: A whole number has the property that when the square of half the number is subtracted from five times the number, we get the number itself.
  - $5x - \left(\frac{x}{2}\right)^2 = x$

Challenge students to write this equation in another form. Engage in a conversation about why they are both correct. For example, when a number is subtracted from itself, the difference is zero. For that reason, the equation above can be written as  $5x - \left(\frac{x}{2}\right)^2 - x = 0$ .

### Example 2 (4 minutes)

Throughout Example 2, have students write their thoughts on personal white boards or a similar tool and show their transcription(s).

- We want to express the following statement using symbolic language: Paulo has a certain amount of money. If he spends \$6.00, then he has  $\frac{1}{4}$  of the original amount left.
- What is the first thing that must be done before we express this situation using symbols?
- We need to define our variables; that is, we must decide what symbol to use and state what it is going to represent.

#### Scaffolding:

Students may need to be reminded that it is not necessary to put the multiplication symbol between a number and a symbol. It is not wrong to include it, but by convention (a common agreement), it is not necessary.



- Suppose we decide to use the symbol  $x$ . We will let  $x$  be the number of dollars Paulo had originally. How do we show Paulo's spending \$6.00 using symbols?
  - To show that Paulo spent \$6.00, we write  $x - 6$ .
- How do we express, "he has  $\frac{1}{4}$  of the original amount?"
  - We can express it as  $\frac{1}{4}x$ .
- Put the parts together to express the following: "Paulo has a certain amount of money. If he spends \$6.00, then he has  $\frac{1}{4}$  of the original amount left." Use  $x$  to represent the amount of money Paulo had originally.
  - $x - 6 = \frac{1}{4}x$

Challenge students to write this equation in another form. Engage in a conversation about why they are both correct. For example, students may decide to show that the six dollars plus what he has left is equal to the amount of money he now has. In symbols,  $x = \frac{1}{4}x + 6$ .

### Example 3 (8 minutes)

Throughout Example 3, have students write their thoughts on personal white boards or a similar tool and show their transcription(s).

- We want to write the following statement using symbolic language: When a fraction of 57 is taken away from 57, what remains exceeds  $\frac{2}{3}$  of 57 by 4.
- The first step is to clearly state what we want our symbol to represent. If we choose the letter  $x$ , then we would say, "Let  $x$  be the fraction of 57" because that is the number that is unknown to us in the written statement. It is acceptable to use any letter to represent the unknown number; but regardless of which letter we use to symbolize the unknown number, we must clearly state what it means. This is called *defining our variables*.
- The hardest part of transcription is figuring out exactly how to write the statement so that it is accurately represented in symbols. Begin with the first part of the statement: "When a fraction of 57 is taken away from 57," how can we capture that information in symbols?
  - Students should write  $57 - x$ .
- How do we write  $\frac{2}{3}$  of 57?
  - If we are trying to find  $\frac{2}{3}$  of 57, then we multiply  $\frac{2}{3} \cdot 57$ .
- Would it be accurate to write  $57 - x = \frac{2}{3} \cdot 57$ ?
  - No, because we are told that "what remains exceeds  $\frac{2}{3}$  of 57 by 4."

#### Scaffolding:

Alternative statement:

- When a number is taken away from 57, what remains is four more than 5 times the number.
- $57 - x = 5x + 4$

#### Scaffolding:

If students have a hard time thinking about these transcriptions, give them something easier to think about. One number, say 10, exceeds another number, say 6, by 4. Is it accurate to represent this by:

$$10 - 4 = 6?$$

$$10 = 6 + 4?$$

$$10 - 6 = 4?$$



- Where does the 4 belong? “What remains exceeds  $\frac{2}{3}$  of 57 by 4.” Think about what the word *exceeds* means in the context of the problem. Specifically, which is bigger:  $57 - x$  or  $\frac{2}{3}$  of 57? How do you know?
  - $57 - x$  is bigger because  $57 - x$  exceeds  $\frac{2}{3}$  of 57 by 4. That means that  $57 - x$  is 4 more than  $\frac{2}{3}$  of 57.
- We know that  $57 - x$  is bigger than  $\frac{2}{3} \cdot 57$  by 4. What would make the two numbers equal?
  - We either have to subtract 4 from  $57 - x$  or add 4 to  $\frac{2}{3} \cdot 57$  to make them equal.
- Now, if  $x$  is the fraction of 57, then we could write  $(57 - x) - 4 = \frac{2}{3} \cdot 57$ , or  $57 - x = \frac{2}{3} \cdot 57 + 4$ . Which is correct?
  - Both transcriptions are correct because both express the written statement accurately.
- Consider this transcription:  $(57 - x) - \frac{2}{3} \cdot 57 = 4$ . Is it an accurate transcription of the information in the written statement?
  - Yes, because  $57 - x$  is bigger than  $\frac{2}{3} \cdot 57$  by 4. That means that the difference between the two numbers is 4. If we subtract the smaller number from the bigger number, we have a difference of 4, and that is what this version of the transcription shows.

#### Example 4 (4 minutes)

Throughout Example 4, have students write their thoughts on personal white boards or a similar tool and show their transcription(s).

- We want to express the following statement using symbolic language: The sum of three consecutive integers is 372.
- Do the first step, and hold up your white board.
  - Let  $x$  be the first integer.
- If we let  $x$  represent the first integer, what do we need to do to get the next consecutive integer?
  - If  $x$  is the first integer, we add 1 to  $x$  to get the next integer.
- In symbols, the next integer would be  $x + 1$ . What do we need to do now to get the next consecutive integer?
  - We need to add 1 to that integer, or  $x + 1 + 1$ ; this is the same as  $x + 2$ .
- Now, express the statement: The sum of three consecutive integers is 372.
  - $x + x + 1 + x + 2 = 372$

#### Scaffolding:

Explain that *consecutive* means one after the next. For example, 18, 19, and 20 are consecutive integers. Consider giving students a number and asking them what the next consecutive integer would be.

Students may also choose to rewrite the above equation as  $3x + 3 = 372$ . Transforming equations such as is a focus of the next few lessons when students begin to solve linear equations. Ask students why both equations are correct.

**Example 5 (3 minutes)**

Throughout Example 5, have students write their thoughts on personal white boards or a similar tool and show their transcription(s).

- We want to express the following statement using symbolic language: The sum of three consecutive odd integers is 93.
- Do the first step, and hold up your white board.
  - *Let  $x$  be the first odd integer.*
- If we let  $x$  represent the first odd integer, what do we need to do to get the next odd integer?
  - *If  $x$  is the first odd integer, we add 2 to  $x$  to get the next odd integer.*
- In symbols, the next odd integer would be  $x + 2$ . What do we need to do now to get the next odd integer?
  - *We need to add 2 to that odd integer, or  $x + 2 + 2$ ; this is the same as  $x + 4$ .*
- Now, express the statement: The sum of three consecutive odd integers is 93.
  - $x + x + 2 + x + 4 = 93$
- Represent the statement “The sum of three consecutive odd integers is 93” in another way. Be prepared to explain why both are correct.
  - *Answers will vary. Accept any correct answers and justifications. For example, students may write  $(x + x + x) + 6 = 93$  and state that it is equivalent to the equation  $x + x + 2 + x + 4 = 93$ . Because the associative and commutative properties of addition were applied, those properties do not change the value of an expression.*

**Scaffolding:**

Consider giving students an odd number and then asking them what the next consecutive odd number is. Then, ask students what they need to add to go from the first number to the second number.

**Exercises (10 minutes)**

Students complete Exercises 1–5 independently or in pairs.

**Exercises**

Write each of the following statements using symbolic language.

1. The sum of four consecutive even integers is  $-28$ .  
*Let  $x$  be the first even integer. Then,  $x + x + 2 + x + 4 + x + 6 = -28$ .*
2. A number is four times larger than the square of half the number.  
*Let  $x$  be the number. Then,  $x = 4 \left(\frac{x}{2}\right)^2$ .*
3. Steven has some money. If he spends \$9.00, then he will have  $\frac{3}{5}$  of the amount he started with.  
*Let  $x$  be the amount of money (in dollars) Steven started with. Then,  $x - 9 = \frac{3}{5}x$ .*
4. The sum of a number squared and three less than twice the number is 129.  
*Let  $x$  be the number. Then,  $x^2 + 2x - 3 = 129$ .*



5. Miriam read a book with an unknown number of pages. The first week, she read five less than  $\frac{1}{3}$  of the pages. The second week, she read 171 more pages and finished the book. Write an equation that represents the total number of pages in the book.

*Let  $x$  be the total number of pages in the book. Then,  $\frac{1}{3}x - 5 + 171 = x$ .*

### Closing (4 minutes)

Summarize, or ask students to summarize, the main points from the lesson:

- We know how to write mathematical statements using symbolic language.
- Written mathematical statements can be represented as more than one correct symbolic statement.
- We must always begin writing a symbolic statement by defining our symbols (variables).
- Complicated statements should be broken into parts or attempted with simple numbers to make the representation in symbolic notation easier.

#### Lesson Summary

Begin all word problems by defining your variables. State clearly what you want each symbol to represent.

Written mathematical statements can be represented as more than one correct symbolic statement.

Break complicated problems into smaller parts, or try working them with simpler numbers.

### Exit Ticket (5 minutes)





## Exit Ticket Sample Solutions

Write each of the following statements using symbolic language.

1. When you square five times a number, you get three more than the number.

*Let  $x$  be the number. Then,  $(5x)^2 = x + 3$ .*

2. Monica had some cookies. She gave seven to her sister. Then, she divided the remainder into two halves, and she still had five cookies left.

*Let  $x$  be the original amount of cookies. Then,  $\frac{1}{2}(x - 7) = 5$ .*

## Problem Set Sample Solutions

Students practice transcribing written statements into symbolic language.

Write each of the following statements using symbolic language.

1. Bruce bought two books. One book costs \$4.00 more than three times the other. Together, the two books cost him \$72.

*Let  $x$  be the cost of the less expensive book. Then,  $x + 4 + 3x = 72$ .*

2. Janet is three years older than her sister Julie. Janet's brother is eight years younger than their sister Julie. The sum of all of their ages is 55 years.

*Let  $x$  be Julie's age. Then,  $(x + 3) + (x - 8) + x = 55$ .*

3. The sum of three consecutive integers is 1,623.

*Let  $x$  be the first integer. Then,  $x + (x + 1) + (x + 2) = 1623$ .*

4. One number is six more than another number. The sum of their squares is 90.

*Let  $x$  be the smaller number. Then,  $x^2 + (x + 6)^2 = 90$ .*

5. When you add 18 to  $\frac{1}{4}$  of a number, you get the number itself.

*Let  $x$  be the number. Then,  $\frac{1}{4}x + 18 = x$ .*

6. When a fraction of 17 is taken away from 17, what remains exceeds one-third of seventeen by six.

*Let  $x$  be the fraction of 17. Then,  $17 - x = \frac{1}{3} \cdot 17 + 6$ .*

7. The sum of two consecutive even integers divided by four is 189.5.

*Let  $x$  be the first even integer. Then,  $\frac{x + (x + 2)}{4} = 189.5$ .*



8. Subtract seven more than twice a number from the square of one-third of the number to get zero.

Let  $x$  be the number. Then,  $\left(\frac{1}{3}x\right)^2 - (2x + 7) = 0$ .

9. The sum of three consecutive integers is 42. Let  $x$  be the middle of the three integers. Transcribe the statement accordingly.

$$(x - 1) + x + (x + 1) = 42$$