



Lesson 13: Angle Sum of a Triangle

Student Outcomes

- Students know the angle sum theorem for triangles; the sum of the interior angles of a triangle is always 180° .
- Students present informal arguments to draw conclusions about the angle sum of a triangle.

Classwork

Concept Development (3 minutes)

- The angle sum theorem for triangles states that the sum of the measures of the interior angles of a triangle is always 180° (\angle sum of Δ).
- It does not matter what kind of triangle it is (i.e., acute, obtuse, right); when the measure of the three angles are added, the sum is always 180° .

Concept Development

$$m\angle 1 + m\angle 2 + m\angle 3 = m\angle 4 + m\angle 5 + m\angle 6 = m\angle 7 + m\angle 8 + m\angle 9 = 180^\circ$$

Note that the sum of the measures of angles 7 and 9 must equal 90° because of the known right angle in the right triangle.

We want to prove that the angle sum of any triangle is 180° . To do so, we use some facts that we already know about geometry:

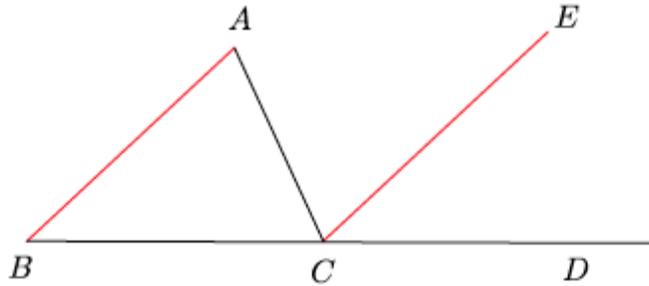
- A straight angle is 180° in measure.
- Corresponding angles of parallel lines are equal in measure (corr. \angle 's, $\overline{AB} \parallel \overline{CD}$).
- Alternate interior angles of parallel lines are equal in measure (alt. \angle 's, $\overline{AB} \parallel \overline{CD}$).

Exploratory Challenge 1 (13 minutes)

Provide students 10 minutes of work time. Once the 10 minutes have passed, review the solutions with students before moving on to Exploratory Challenge 2.

Exploratory Challenge 1

Let triangle ABC be given. On the ray from B to C , take a point D so that C is between B and D . Through point C , draw a segment parallel to \overline{AB} , as shown. Extend the segments AB and CE . Line AC is the transversal that intersects the parallel lines.



- a. Name the three interior angles of triangle ABC .

$\angle ABC, \angle BAC, \angle BCA$

- b. Name the straight angle.

$\angle BCD$

Our goal is to show that the measures of the three interior angles of triangle ABC are equal to the measures of the angles that make up the straight angle. We already know that a straight angle is 180° in measure. If we can show that the interior angles of the triangle are the same as the angles of the straight angle, then we will have proven that the sum of the measures of the interior angles of the triangle have a sum of 180° .

- c. What kinds of angles are $\angle ABC$ and $\angle ECD$? What does that mean about their measures?

$\angle ABC$ and $\angle ECD$ are corresponding angles. Corresponding angles of parallel lines are equal in measure (corr. \angle 's, $\overline{AB} \parallel \overline{CE}$).

- d. What kinds of angles are $\angle BAC$ and $\angle ECA$? What does that mean about their measures?

$\angle BAC$ and $\angle ECA$ are alternate interior angles. Alternate interior angles of parallel lines are equal in measure (alt. \angle 's, $\overline{AB} \parallel \overline{CE}$).

- e. We know that $m\angle BCD = m\angle BCA + m\angle ECA + m\angle ECD = 180^\circ$. Use substitution to show that the measures of the three interior angles of the triangle have a sum of 180° .

$m\angle BCD = m\angle BCA + m\angle BAC + m\angle ABC = 180^\circ$ (\angle sum of Δ)

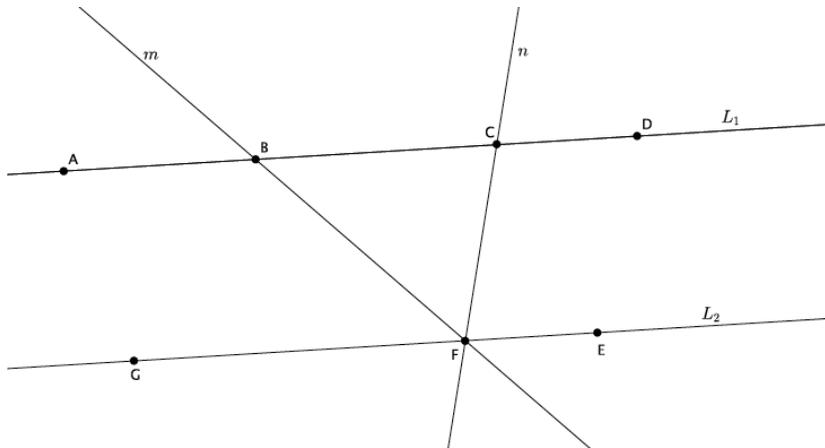
MP.3

Exploratory Challenge 2 (20 minutes)

Provide students 15 minutes of work time. Once the 15 minutes have passed, review the solutions with students.

Exploratory Challenge 2

The figure below shows parallel lines L_1 and L_2 . Let m and n be transversals that intersect L_1 at points B and C , respectively, and L_2 at point F , as shown. Let A be a point on L_1 to the left of B , D be a point on L_1 to the right of C , G be a point on L_2 to the left of F , and E be a point on L_2 to the right of F .



- a. Name the triangle in the figure.

$\triangle BCF$

- b. Name a straight angle that will be useful in proving that the sum of the measures of the interior angles of the triangle is 180° .

$\angle GFE$

As before, our goal is to show that the sum of the measures of the interior angles of the triangle are equal to the measure of the straight angle. Use what you learned from Exploratory Challenge 1 to show that the measures of the interior angles of a triangle have a sum of 180° .

- c. Write your proof below.

The straight angle $\angle GFE$ is comprised of $\angle GFB$, $\angle BFC$, and $\angle EFC$. Alternate interior angles of parallel lines are equal in measure (alt. \angle 's, $\overline{AD} \parallel \overline{CE}$). For that reason, $\angle BCF = \angle EFC$ and $\angle CBF = \angle GFB$. Since $\angle GFE$ is a straight angle, it is equal to 180° . Then, $\angle GFE = \angle GFB + \angle BFC + \angle EFC = 180^\circ$.

By substitution, $\angle GFE = \angle CBF + \angle BFC + \angle BCF = 180^\circ$. Therefore, the sum of the measures of the interior angles of a triangle is 180° (\angle sum of \triangle).

MP.3

Closing (4 minutes)

Summarize, or have students summarize, the lesson.

- All triangles have interior angles whose measures sum to 180° .
- We can prove that the sum of the measures of the interior angles of a triangle are equal to the measure of a straight angle using what we know about alternate interior angles and corresponding angles of parallel lines.

Lesson Summary

All triangles have a sum of measures of the interior angles equal to 180° .

The proof that a triangle has a sum of measures of the interior angles equal to 180° is dependent upon the knowledge of straight angles and angle relationships of parallel lines cut by a transversal.

Exit Ticket (5 minutes)

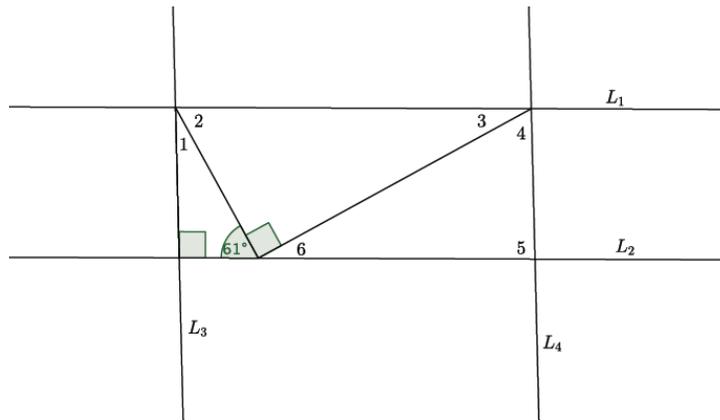
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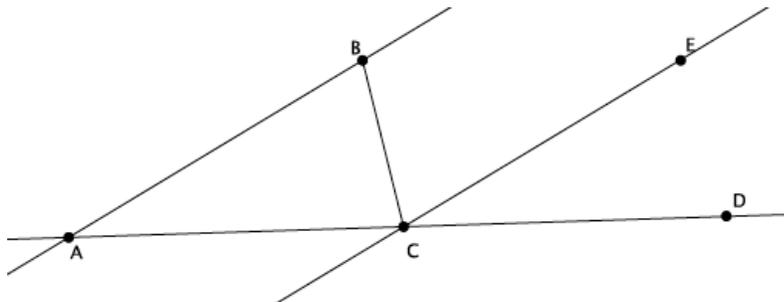
Lesson 13: Angle Sum of a Triangle

Exit Ticket

1. If $L_1 \parallel L_2$, and $L_3 \parallel L_4$, what is the measure of $\angle 1$? Explain how you arrived at your answer.

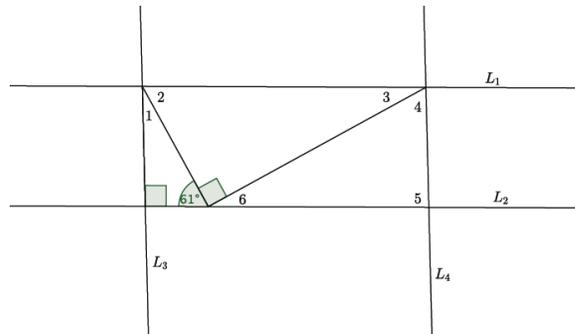


2. Given that line AB is parallel to line CE , present an informal argument to prove that the measures of the interior angles of triangle ABC have a sum of 180° .



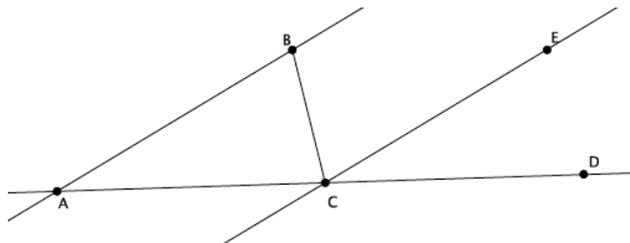
Exit Ticket Sample Solutions

1. If $L_1 \parallel L_2$, and $L_3 \parallel L_4$, what is the measure of $\angle 1$? Explain how you arrived at your answer.



The measure of angle 1 is 29° . I know that the angle sum of triangles is 180° . I already know that two of the angles of the triangle are 90° and 61° .

2. Given that line AB is parallel to line CE , present an informal argument to prove that the measures of the interior angles of triangle ABC have a sum of 180° .

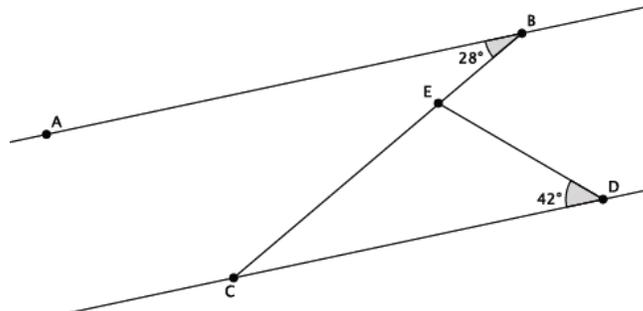


Since \overline{AB} is parallel to \overline{CE} , the corresponding angles $\angle BAC$ and $\angle ECD$ are equal in measure. Similarly, $\angle ABC$ and $\angle ECB$ are equal in measure because they are alternate interior angles. Since $\angle ACD$ is a straight angle (i.e., equal to 180° in measure), substitution shows that triangle ABC has a sum of 180° . Specifically, the straight angle is made up of $\angle ACB$, $\angle ECB$, and $\angle ECD$. $\angle ACB$ is one of the interior angles of the triangle and one of the angles of the straight angle. We know that $\angle ABC$ has the same measure as $\angle ECB$ and that $\angle BAC$ has the same measure as $\angle ECD$. Therefore, the sum of the measures of the interior angles will be the same as the sum of the measures of the angles of the straight angle, which is 180° .

Problem Set Sample Solutions

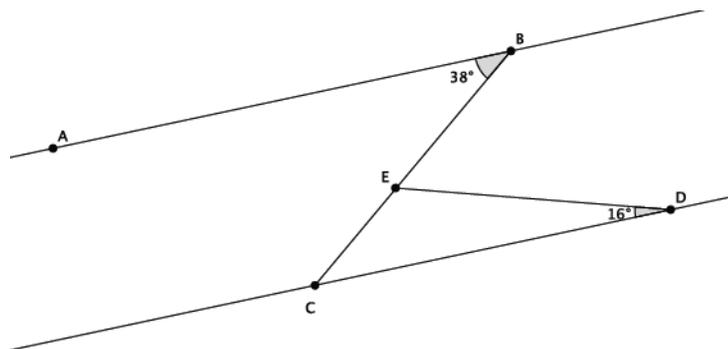
Students practice presenting informal arguments about the sum of the angles of a triangle using the theorem to find the measures of missing angles.

1. In the diagram below, line AB is parallel to line CD , that is, $L_{AB} \parallel L_{CD}$. The measure of $\angle ABC$ is 28° and the measure of $\angle EDC$ is 42° . Find the measure of $\angle CED$. Explain why you are correct by presenting an informal argument that uses the angle sum of a triangle.



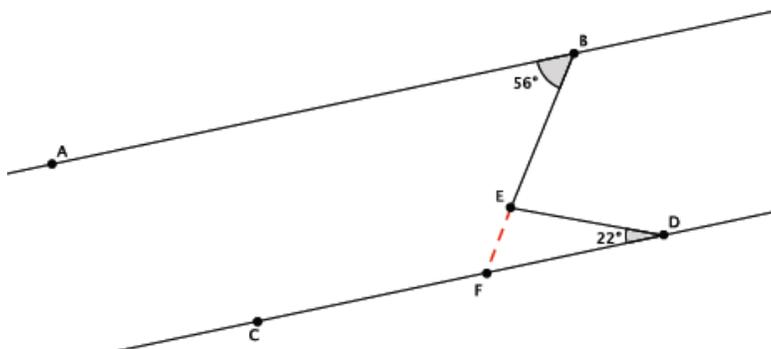
The measure of $\angle CED$ is 110° . This is the correct measure for the angle because $\angle ABC$ and $\angle DCE$ are alternate interior angles of parallel lines. That means that the angles are congruent and have the same measure. Since the angle sum of a triangle is 180° , then $\angle CED = 180^\circ - (28^\circ + 42^\circ)$ and $\angle CED$ is 110° .

2. In the diagram below, line AB is parallel to line CD , that is, $L_{AB} \parallel L_{CD}$. The measure of $\angle ABE$ is 38° , and the measure of $\angle EDC$ is 16° . Find the measure of $\angle BED$. Explain why you are correct by presenting an informal argument that uses the angle sum of a triangle. (Hint: Find the measure of $\angle CED$ first, and then use that measure to find the measure of $\angle BED$.)



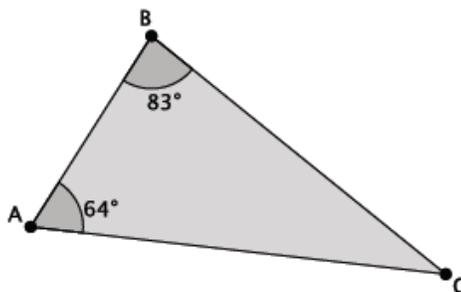
The measure of $\angle BED$ is 54° . This is the correct measure for the angle because $\angle ABC$ and $\angle DCE$ are alternate interior angles of parallel lines. That means that the angles are congruent and have the same measure. Since the angle sum of a triangle is 180° , then $\angle CED = 180^\circ - (38^\circ + 16^\circ) = 126^\circ$. The straight angle $\angle BEC$ is made up of $\angle CED$ and $\angle BED$. Since we know straight angles measure 180° and $\angle CED = 126^\circ$, then $\angle BED$ is 54° .

3. In the diagram below, line AB is parallel to line CD , that is, $L_{AB} \parallel L_{CD}$. The measure of $\angle ABE$ is 56° , and the measure of $\angle EDC$ is 22° . Find the measure of $\angle BED$. Explain why you are correct by presenting an informal argument that uses the angle sum of a triangle. (Hint: Extend the segment BE so that it intersects line CD .)



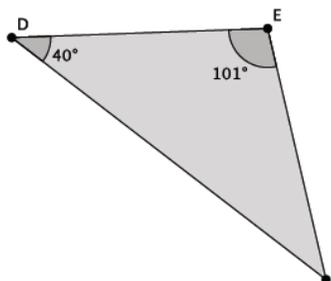
The measure of $\angle BED$ is 78° . This is the correct measure for the angle because $\angle ABE$ and $\angle DFE$ are alternate interior angles of parallel lines. That means that the angles are congruent and have the same measure. Since the angle sum of a triangle is 180° , then $\angle FED = 180^\circ - (56^\circ + 22^\circ) = 102^\circ$. The straight angle $\angle BEF$ is made up of $\angle FED$ and $\angle BED$. Since straight angles measure 180° and $\angle FED$ is 102° , then $\angle BED$ is 78° .

4. What is the measure of $\angle ACB$?



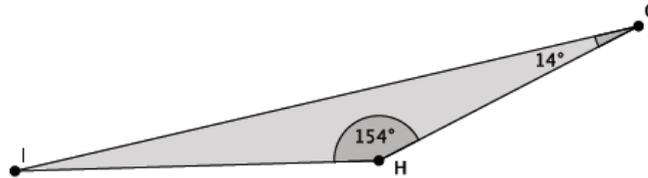
The measure of $\angle ACB$ is $180^\circ - (83^\circ + 64^\circ)$, which is equal to 33° .

5. What is the measure of $\angle EFD$?



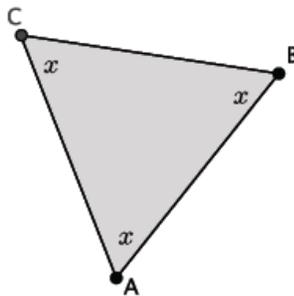
The measure of $\angle EFD$ is $180^\circ - (101^\circ + 40^\circ)$, which is equal to 39° .

6. What is the measure of $\angle HIG$?



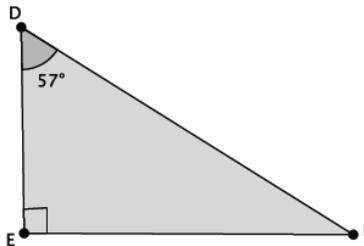
The measure of $\angle HIG$ is $180^\circ - (154^\circ + 14^\circ)$, which is equal to 12° .

7. What is the measure of $\angle ABC$?



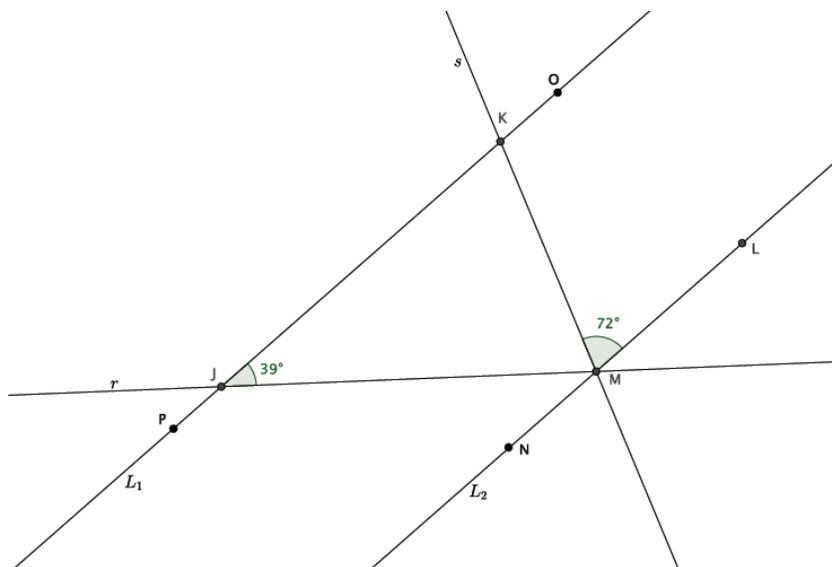
The measure of $\angle ABC$ is 60° because $60^\circ + 60^\circ + 60^\circ$, which is equal to 180° .

8. Triangle DEF is a right triangle. What is the measure of $\angle EFD$?



The measure of $\angle EFD$ is $90^\circ - 57^\circ$, which is equal to 33° .

9. In the diagram below, Lines L_1 and L_2 are parallel. Transversals r and s intersect both lines at the points shown below. Determine the measure of $\angle JMK$. Explain how you know you are correct.



The Lines L_1 and L_2 are parallel, which means that the alternate interior angles formed by the transversals are equal. Specifically, $\angle LMK = \angle JKM = 72^\circ$. Since $\triangle JKM$ has a sum of interior angles equal to 180° , $\angle KJM + \angle JMK + \angle JKM = 180^\circ$. By substitution, we have $39^\circ + \angle JMK + 72^\circ = 180^\circ$; therefore, $\angle JMK$ is 69° .