Lesson 12: Choice of Unit

Student Outcomes

- Students understand how choice of unit determines how easy or difficult it is to understand an expression of measurement.
- Students determine appropriate units for various measurements and rewrite measurements based on new units.

Lesson Notes

This lesson focuses on choosing appropriate units. It is important for students to see the simple example (i.e., dining table measurements), as well as more sophisticated examples from physics and astronomy. We want students to understand the *necessity of learning to read, write, and operate in scientific notation*. For this very reason, we provide real data and explanations for why scientific notation is important and necessary in advanced sciences. This is a challenging, but crucial, lesson and should not be skipped.

Classwork

MP.4

Concept Development (2 minutes): The main reason for using scientific notation is to sensibly and efficiently record and convey the results of measurements. When we use scientific notation, the question of what unit to use naturally arises. In everyday context, this issue is easy to understand. For example, suppose we want to measure the horizontal dimensions of a dining table. In this case, measurements of 42×60 sq. in., or for that matter, $3\frac{1}{2} \times 5$ sq. ft. are commonly accepted. However, when the same measurement is presented as

$$\frac{0.7}{1056} \times \frac{1}{1056} \text{ sq. mi.},$$

it is confusing because we cannot relate a unit as long as a mile to a space as small as a dining room (*recall:* 1 mile is 5,280 feet), not to mention that the numbers $\frac{0.7}{1056}$ and $\frac{1}{1056}$ are unmanageable.

Exercises 1–3 (5 minutes)

Have students complete Exercises 1–3 in small groups.

Exercise 1

A certain brand of MP3 player will display how long it will take to play through its entire music library. If the maximum number of songs the MP3 player can hold is $1,000$ (and the average song length is 4 minutes), would you want the time displayed in terms of seconds-, days-, or years-worth of music? Explain.

It makes the most sense to have the time displayed in days because numbers such as $240,000$ seconds-worth of music and $\frac{5}{657}$ of a year are more difficult to understand than about 2.8 days.

Exercise 2

You have been asked to make frosted cupcakes to sell at a school fundraiser. Each frosted cupcake contains about 20 grams of sugar. Bake sale coordinators expect 500 people will attend the event. Assume everyone who attends will buy a cupcake; does it make sense to buy sugar in grams, pounds, or tons? Explain.

Because each cupcake contains about 20 grams of sugar, we will need 500×20 grams of sugar. Therefore, grams are too small of a measurement, while tons are too large. Therefore, the sugar should be purchased in pounds.

Exercise 3

The seafloor spreads at a rate of approximately 10 cm per year. If you were to collect data on the spread of the seafloor each week, which unit should you use to record your data? Explain.

The seafloor spreads 10 cm per year, which is less than 1 cm per month. Data will be collected each week, so it makes the most sense to measure the spread with a unit like millimeters.

Example 1 (3 minutes)

Now let's look at the field of particle physics or the study of subatomic particles, such as protons, electrons, neutrons, and mesons. In the previous lesson, we worked with the masses of protons and electrons, which are

1.672622×10^{-27} and $9.10938291 \times 10^{-31}$ kg, respectively.

The factors 10^{-27} and 10^{-31} suggest that we are dealing with very small quantities; therefore, the use of a unit other than kilograms may be necessary. Should we use gram instead of kilogram? At first glance, yes, but when we do, we get the numbers 1.672622×10^{-24} g and $9.10938291 \times 10^{-28}$ g. One cannot claim that these are much easier to deal with.

> *Scaffolding:*
> Remind students that a kilogram is $1{,}000$ grams, so we can quickly write the masses in the new unit using our knowledge of powers of 10.

- Is it easier to visualize something that is 10^{-24} compared to 10^{-27}?
  - *Of course not. That is why a better unit, the gigaelectronvolt, is used.*

MP.2

For this and other reasons, particle physicists use the **gigaelectronvolt**, $\frac{\mathbf{GeV}}{\boldsymbol{c^2}}$ as a unit of mass:

$$1\frac{\text{GeV}}{c^2} = 1.783 \times 10^{-27} \text{ kg}.$$

The gigaelectronvolt, $\frac{\mathbf{GeV}}{\boldsymbol{c^2}}$, is what particle physicists use for a unit of mass.

1 gigaelectronvolt = 1.783×10^{-27} kg

Mass of 1 proton = $1.672\,622 \times 10^{-27}$ kg

MP.2

The very name of the unit gives a hint that it was created for a purpose, but we do not need to explore that at this time. The important piece of information is to understand that 1.783×10^{-27} kg is a unit, and it represents 1 gigaelectronvolt. Thus, the mass of a proton is $0.938 \frac{\text{GeV}}{c^2}$ rounded to the nearest 10^{-3}, and the mass of an electron is $0.000511 \frac{\text{GeV}}{c^2}$ rounded to the nearest 10^{-6}. A justification[1] for this new unit is that the masses of a large class of subatomic particles have the same order of magnitude as 1 gigaelectronvolt.

Exercise 4 (3 minutes)

Have students complete Exercise 4 independently.

Exercise 4

Show that the mass of a proton is $0.938 \frac{\text{GeV}}{c^2}$.

Let x represent the number of gigaelectronvolts equal to the mass of a proton.

$$x\left(\frac{\text{GeV}}{c^2}\right) = \textit{mass of proton}$$
$$x(1.783 \times 10^{-27}) = 1.672622 \times 10^{-27}$$
$$x = \frac{1.672622 \times 10^{-27}}{1.783 \times 10^{-27}}$$
$$= \frac{1.672622}{1.783}$$
$$\approx 0.938$$

Example 2 (4 minutes)

Choosing proper units is also essential for work with very large numbers, such as those involved in astronomy (e.g., astronomical distances). The distance from the sun to the nearest star (Proxima Centauri) is approximately

$$4.013\,336\,473 \times 10^{13} \text{ km}.$$

In 1838, F.W. Bessel[2] was the first to measure the distance to a star, 61 Cygni, and its distance from the sun was

$$1.078\,807 \times 10^{14} \text{ km}.$$

MP.2

For numbers of this magnitude, we need to use a unit other than kilometers.

In popular science writing, a commonly used unit is the light-year, or the <u>distance</u> light travels in one year (note: one year is defined as 365.25 days).

$$\textbf{1 light-year} = 9{,}460{,}730{,}472{,}580.800 \text{ km} \approx 9.46073 \times 10^{12} \text{ km}$$

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[1] There are other reasons coming from considerations within physics.

[2] Students will discover *Bessel functions* if they pursue STEM subjects at universities. We now know that 61 Cygni is actually a binary system consisting of *two* stars orbiting each other around a point called their *center of gravity*, but Bessel did not have a sufficiently powerful telescope to resolve the binary system.

One light-year is approximately 9.46073×10^{12} km. Currently, the distance of Proxima Centauri to the sun is approximately 4.2421 light-years, and the distance of 61 Cygni to the sun is approximately 11.403 light-years. When you ignore that 10^{12} is an enormous number, it is easier to think of the distances as 4.2 light-years and 11.4 light-years for these stars. For example, we immediately see that 61 Cygni is almost 3 times further from the sun than Proxima Centauri.

To measure the distance of stars in our galaxy, the light-year is a logical unit to use. Since launching the powerful Hubble Space Telescope in 1990, galaxies billions of light-years from the sun have been discovered. For these galaxies, the **gigalight-year** (or 10^9 light-years) is often used.

Exercise 5 (3 minutes)

Have students work on Exercise 5 independently.

Exercise 5

The distance of the nearest star (Proxima Centauri) to the sun is approximately $4.013\,336\,473 \times 10^{13}$ km. Show that Proxima Centauri is 4.2421 light-years from the sun.

Let x represent the number of light-years Proxima Centauri is from the sun.

$$x(9.46073 \times 10^{12}) = 4.013336473 \times 10^{13}$$
$$x = \frac{4.013336473 \times 10^{13}}{9.46073 \times 10^{12}}$$
$$= \frac{4.013336473}{9.46073} \times 10$$
$$= 0.424210021 \times 10$$
$$\approx 4.2421$$

Exploratory Challenge 1 (8 minutes)

Finally, let us look at an example involving the masses of the planets in our solar system. They range from Mercury's 3.3022×10^{23} kg to Jupiter's 1.8986×10^{27} kg. However, Earth's mass is the fourth heaviest among the eight planets[3], and it seems reasonable to use it as the point of reference for discussions among planets. Therefore, a new unit is M_E, the mass of the Earth, or 5.97219×10^{24} kg.

Suggested white-board activity: Show students the table below, leaving the masses for Mercury and Jupiter blank. Demonstrate how to rewrite the mass of Mercury in terms of the new unit, M_E. Then, have students rewrite the mass of Jupiter using the new unit. Finally, complete the chart with the rewritten masses.

[3]Since 2006, Pluto is no longer classified as a planet. If Pluto was still considered a planet, then Earth would be the fifth-heaviest planet, right in the middle, which would further boost Earth's claim to be the point of reference.

Mercury: Let x represent the mass of Mercury in the unit M_E. We want to determine what number times the new unit is equal to the mass of Mercury in kilograms. Since $M_E = 5.97219 \times 10^{24}$, then:

$$(5.97219 \times 10^{24})x = 3.3022 \times 10^{23}$$
$$x = \frac{3.3022 \times 10^{23}}{5.97219 \times 10^{24}}$$
$$= \frac{3.3022}{5.97219} \times \frac{10^{23}}{10^{24}}$$
$$\approx 0.553 \times 10^{-1}$$
$$= 0.0553.$$

Mercury's mass is $0.0553\ M_E$.

Jupiter: Let x represent the mass of Jupiter in the unit M_E. We want to determine what number times the new unit is equal to the mass of Jupiter in kilograms. Since $M_E = 5.97219 \times 10^{24}$, then:

$$(5.97219 \times 10^{24})x = 1.8986 \times 10^{27}$$
$$x = \frac{1.8986 \times 10^{27}}{5.97219 \times 10^{24}}$$
$$= \frac{1.8986}{5.97219} \times \frac{10^{27}}{10^{24}}$$
$$\approx 0.318 \times 10^{3}$$
$$= 318.$$

Jupiter's mass is $318\ M_E$.

Mercury	$\mathbf{0.0553\ M_E}$	Jupiter	$\mathbf{318\ M_E}$
Venus	$0.815\ M_E$	Saturn	$95.2\ M_E$
Earth	$1\ M_E$	Uranus	$14.5\ M_E$
Mars	$0.107\ M_E$	Neptune	$17.2\ M_E$

Exploratory Challenge 2/Exercises 6–8 (10 minutes)

Have students complete Exercises 6–8 independently or in small groups. Allow time for groups to discuss their choice of unit and the reasoning for choosing it.

Exploratory Challenge 2

Suppose you are researching atomic diameters and find that credible sources provided the diameters of five different atoms as shown in the table below. All measurements are in centimeters.

1×10^{-8}	1×10^{-12}	5×10^{-8}	5×10^{-10}	5.29×10^{-11}

Exercise 6

What new unit might you introduce in order to discuss the differences in diameter measurements?

There are several answers that students could give for their choice of unit. Accept any reasonable answer, provided the explanation is clear and correct. Some students may choose 10^{-12} as their unit because all measurements could then be expressed without exponential notation. Other students may decide that 10^{-8} should be the unit because two measurements are already of that order of magnitude. Still, other students may choose 10^{-10} because that is the average of the powers.

Exercise 7

Name your unit, and explain why you chose it.

Students can name their unit anything reasonable, as long as they clearly state what their unit is and how it will be written. For example, if a student chooses a unit of 10^{-10}, then he or she should state that the unit will be represented with a letter. For example, Y, then $Y = 10^{-10}$.

Exercise 8

Using the unit you have defined, rewrite the five diameter measurements.

Using the unit $Y = 10^{-10}$, then:

$1\times10^{-8} = 100\ Y$	$1\times10^{-12} = 0.01\ Y$	$5\times10^{-8} = 500\ Y$	$5\times10^{-10} = 5\ Y$	$5.29\times10^{-11} = 0.529\ Y$

Closing (2 minutes)

Summarize the lesson:

- Choosing an appropriate unit allows us to determine the size of the numbers we are dealing with.
  For example, the dining table measurement:

$$42 \times 60 \text{ sq. in.} = 3\frac{1}{2} \times 5 \text{ sq. ft.} = \frac{0.7}{1056} \times \frac{1}{1056} \text{ sq. mi.}$$

- We have reinforced their ability to read, write, and operate with numbers in scientific notation.

Exit Ticket (5 minutes)

Name ______________________________ Date ______________

Lesson 12: Choice of Unit

Exit Ticket

1. The table below shows an approximation of the national debt at the beginning of each decade over the last century. Choose a unit that would make a discussion about the growth of the national debt easier. Name your unit, and explain your choice.

Year	Debt in Dollars
1900	2.1×10^9
1910	2.7×10^9
1920	2.6×10^{10}
1930	1.6×10^{10}
1940	4.3×10^{10}
1950	2.6×10^{11}
1960	2.9×10^{11}
1970	3.7×10^{11}
1980	9.1×10^{11}
1990	3.2×10^{12}
2000	5.7×10^{12}

2. Using the new unit you have defined, rewrite the debt for years 1900, 1930, 1960, and 2000.

Exit Ticket Sample Solutions

1. **The table below shows an approximation of the national debt at the beginning of each decade over the last century. Choose a unit that would make a discussion about the growth of the national debt easier. Name your unit, and explain your choice.**

   Students will likely choose 10^{11} as their unit because the majority of the data is of that magnitude. Accept any reasonable answer that students provide. Verify that they have named their unit.

Year	Debt in Dollars
1900	2.1×10^9
1910	2.7×10^9
1920	2.6×10^{10}
1930	1.6×10^{10}
1940	4.3×10^{10}
1950	2.6×10^{11}
1960	2.9×10^{11}
1970	3.7×10^{11}
1980	9.1×10^{11}
1990	3.2×10^{12}
2000	5.7×10^{12}

2. **Using the new unit you have defined, rewrite the debt for years 1900, 1930, 1960, and 2000.**

   Let D represent the unit 10^{11}. Then, the debt in 1900 is $0.021D$, in 1930 it is $0.16D$, in 1960 it is $2.9D$, and $57D$ in 2000.

Problem Set Sample Solution

1. **Verify the claim that, in terms of gigaelectronvolts, the mass of an electron is 0.000511.**

   Let x represent the number of gigaelectronvolts equal to the mass of an electron.

$$x\left(\frac{\text{GeV}}{c^2}\right) = \textit{Mass of electron}$$

$$x(1.783 \times 10^{-27}) = 9.109\,382\,91 \times 10^{-31}$$

$$x = \frac{9.109\,382\,91 \times 10^{-31} \times 10^{31}}{1.783 \times 10^{-27} \times 10^{31}}$$

$$= \frac{9.109\,382\,91}{1.783 \times 10^4}$$

$$= \frac{9.109\,382\,91}{17\,830}$$

$$\approx 0.000\,511$$

2. The maximum distance between Earth and the sun is 1.52098232×10^8 km, and the minimum distance is 1.47098290×10^8 km.[4] What is the average distance between Earth and the sun in scientific notation?

$$\textit{average distance} = \frac{1.52098232 \times 10^8 + 1.47098290 \times 10^8}{2}$$
$$= \frac{(1.52098232 + 1.47098290) \times 10^8}{2}$$
$$= \frac{2.99196522 \times 10^8}{2}$$
$$= \frac{2.99196522}{2} \times 10^8$$
$$= 1.49598261 \times 10^8 \text{km}$$

3. Suppose you measure the following masses in terms of kilograms:

2.6×10^{21}	9.04×10^{23}
8.82×10^{23}	2.3×10^{18}
1.8×10^{12}	2.103×10^{22}
8.1×10^{20}	6.23×10^{18}
6.723×10^{19}	1.15×10^{20}
7.07×10^{21}	7.210×10^{29}
5.11×10^{25}	7.35×10^{24}
7.8×10^{19}	5.82×10^{26}

What new unit might you introduce in order to aid discussion of the masses in this problem? Name your unit, and express it using some power of 10. Rewrite each number using your newly defined unit.

A very motivated student may search the Internet and find that units exist that convert large masses to reasonable numbers, such as petagrams (10^{12} kg), exagrams (10^{15} kg), or zetagrams (10^{18} kg). More likely, students will decide that something near 10^{20} should be used as a unit because many of the numbers are near that magnitude. There is one value, 1.8×10^{12}, that serves as an outlier and should be ignored because it is much smaller than the majority of the data. Students can name their unit anything reasonable. The answers provided are suggestions, but any reasonable answers should be accepted.

Let U be defined as the unit 10^{20}.

$2.6 \times 10^{21} = 26U$	$9.04 \times 10^{23} = 9040U$
$8.82 \times 10^{23} = 8820U$	$2.3 \times 10^{18} = 0.023U$
$1.8 \times 10^{12} = 0.000\,000\,018U$	$2.103 \times 10^{22} = 210.3U$
$8.1 \times 10^{0} = 8.1U$	$6.23 \times 10^{18} = 0.0623U$
$6.723 \times 10^{19} = 0.6723U$	$1.15 \times 10^{20} = 1.15U$
$7.07 \times 10^{21} = 70.7U$	$7.210 \times 10^{29} = 7\,210\,000\,000U$
$5.11 \times 10^{25} = 511\,000U$	$7.35 \times 10^{24} = 73\ 500U$
$7.8 \times 10^{19} = 0.78U$	$5.82 \times 10^{26} = 5\,820\,000U$

[4]Note: Earth's orbit is elliptical, not circular.