



Lesson 8: Estimating Quantities

Student Outcomes

- Students compare and estimate quantities in the form of a single digit times a power of 10.
- Students use their knowledge of ratios, fractions, and laws of exponents to simplify expressions.

Classwork

Discussion (1 minute)

Now that we know about positive and negative powers of 10, we can compare numbers and estimate how many times greater one quantity is compared to another. Note that in the first and subsequent examples when we compare two values a and b , we immediately write the value of the ratio $\frac{a}{b}$ as opposed to writing the ratio $a:b$ first.

With our knowledge of the laws of integer exponents, we can also do other computations to estimate quantities.

Example 1 (4 minutes)

In 1723, the population of New York City was approximately 7,248. By 1870, almost 150 years later, the population had grown to 942,292. We want to determine approximately how many times greater the population was in 1870 compared to 1723.

The word *approximately* in the question lets us know that we do not need to find a precise answer, so we approximate both populations as powers of 10.

- Population in 1723: $7248 < 9999 < 10000 = 10^4$
- Population in 1870: $942\,292 < 999\,999 < 1\,000\,000 = 10^6$

We want to compare the population in 1870 to the population in 1723:

$$\frac{10^6}{10^4}$$

Now we can use what we know about the laws of exponents to simplify the expression and answer the question:

$$\frac{10^6}{10^4} = 10^2.$$

Therefore, there were approximately 100 times more people in New York City in 1870 compared to 1723.

Exercise 1 (3 minutes)

Have students complete Exercise 1 independently.

Exercise 1

The Federal Reserve states that the average household in January of 2013 had \$7,122 in credit card debt. About how many times greater is the U.S. national debt, which is \$16,755,133,009,522? Rewrite each number to the nearest power of 10 that exceeds it, and then compare.

Household debt = $7122 < 9999 < 10000 = 10^4$.

U.S. debt = $16\,755\,133\,009\,522 < 99\,999\,999\,999\,999 < 100\,000\,000\,000\,000 = 10^{14}$.

$\frac{10^{14}}{10^4} = 10^{14-4} = 10^{10}$. *The U.S. national debt is 10^{10} times greater than the average household's credit card debt.*

Discussion (3 minutes)

If our calculations were more precise in the last example, we would have seen that by 1870 the population of New York City actually increased by about 130 times from what it was in 1723.

In order to be more precise, we need to use estimations of our original numbers that are more precise than just powers of 10.

For example, instead of estimating the population of New York City in 1723 (7,248 people) to be 10^4 , we can use a more precise estimation: 7×10^3 . Using a single-digit integer times a power of ten is more precise because we are rounding the population to the nearest thousand. Conversely, using only a power of ten, we are rounding the population to the nearest ten thousand.

Consider that the actual population is 7,248.

- $10^4 = 10000$
- $7 \times 10^3 = 7 \times 1000 = 7000$

Which of these two estimations is closer to the actual population?

Clearly, 7×10^3 is a more precise estimation.

Scaffolding:

- Make it clear to students that more precise estimations lead to more precise answers.
- In a later lesson, we learn how to be even more precise than in Example 2.

Example 2 (4 minutes)

Let's compare the population of New York City to the population of New York State. Specifically, let's find out how many times greater the population of New York State is compared to that of New York City.

The population of New York City is 8,336,697. Let's round this number to the nearest million; this gives us 8,000,000. Written as single-digit integer times a power of 10:

$$8\,000\,000 = 8 \times 10^6.$$

The population of New York State is 19,570,261. Rounding to the nearest million gives us 20,000,000. Written as a single-digit integer times a power of 10:

$$20\,000\,000 = 2 \times 10^7.$$



To estimate the difference in size we compare state population to city population:

$$\frac{2 \times 10^7}{8 \times 10^6}$$

Now we simplify the expression to find the answer:

$$\begin{aligned} \frac{2 \times 10^7}{8 \times 10^6} &= \frac{2}{8} \times \frac{10^7}{10^6} && \text{By the product formula} \\ &= \frac{1}{4} \times 10 && \text{By equivalent fractions and the first law of exponents} \\ &= 0.25 \times 10 \\ &= 2.5 \end{aligned}$$

Therefore, the population of the state is 2.5 times that of the city.

Example 3 (4 minutes)

There are about 9 billion devices connected to the Internet. If a wireless router can support 300 devices, about how many wireless routers are necessary to connect all 9 billion devices wirelessly?

Because 9 billion is a very large number, we should express it as a single-digit integer times a power of 10.

$$9\,000\,000\,000 = 9 \times 10^9$$

The laws of exponents tells us that our calculations will be easier if we also express 300 as a single-digit integer times a power of 10, even though 300 is much smaller.

$$300 = 3 \times 10^2$$

We want to know how many wireless routers are necessary to support 9 billion devices, so we must divide

$$\frac{9 \times 10^9}{3 \times 10^2}$$

Now, we can simplify the expression to find the answer:

$$\begin{aligned} \frac{9 \times 10^9}{3 \times 10^2} &= \frac{9}{3} \times \frac{10^9}{10^2} && \text{By the product formula} \\ &= 3 \times 10^7 && \text{By equivalent fractions and the first law of exponents} \\ &= 30\,000\,000 \end{aligned}$$

About 30 million routers are necessary to connect all devices wirelessly.



Exercises 2–4 (5 minutes)

Have students complete Exercises 2–4 independently or in pairs.

Exercise 2

There are about 3,000,000 students attending school, kindergarten through Grade 12, in New York. Express the number of students as a single-digit integer times a power of 10.

$$3\,000\,000 = 3 \times 10^6$$

The average number of students attending a middle school in New York is 8×10^2 . How many times greater is the overall number of K–12 students compared to the average number of middle school students?

$$\begin{aligned} \frac{3 \times 10^6}{8 \times 10^2} &= \frac{3}{8} \times \frac{10^6}{10^2} \\ &= \frac{3}{8} \times 10^4 \\ &= 0.375 \times 10^4 \\ &= 3750 \end{aligned}$$

There are about 3,750 times more students in K–12 compared to the number of students in middle school.

Exercise 3

A conservative estimate of the number of stars in the universe is 6×10^{22} . The average human can see about 3,000 stars at night with his naked eye. About how many times more stars are there in the universe compared to the stars a human can actually see?

$$\frac{6 \times 10^{22}}{3 \times 10^3} = \frac{6}{3} \times \frac{10^{22}}{10^3} = 2 \times 10^{22-3} = 2 \times 10^{19}$$

There are about 2×10^{19} times more stars in the universe compared to the number we can actually see.

Exercise 4

The estimated world population in 2011 was 7×10^9 . Of the total population, 682 million of those people were left-handed. Approximately what percentage of the world population is left-handed according to the 2011 estimation?

$$682\,000\,000 \approx 700\,000\,000 = 7 \times 10^8$$

$$\begin{aligned} \frac{7 \times 10^8}{7 \times 10^9} &= \frac{7}{7} \times \frac{10^8}{10^9} \\ &= 1 \times \frac{1}{10} \\ &= \frac{1}{10} \end{aligned}$$

About one-tenth of the population is left-handed, which is equal to 10%.



Example 4 (3 minutes)

The average American household spends about \$40,000 each year. If there are about 1×10^8 households, what is the total amount of money spent by American households in one year?

Let's express \$40,000 as a single-digit integer times a power of 10.

$$40000 = 4 \times 10^4$$

The question asks us how much money *all* American households spend in one year, which means that we need to multiply the amount spent by one household by the total number of households:

$$(4 \times 10^4)(1 \times 10^8) = (4 \times 1)(10^4 \times 10^8) \quad \text{By repeated use of associative and commutative properties}$$

$$= 4 \times 10^{12} \quad \text{By the first law of exponents}$$

Therefore, American households spend about \$4,000,000,000,000 each year altogether!

Exercise 5 (2 minutes)

Have students complete Exercise 5 independently.

Exercise 5

The average person takes about 30,000 breaths per day. Express this number as a single-digit integer times a power of 10.

$$30000 = 3 \times 10^4$$

If the average American lives about 80 years (or about 30,000 days), how many total breaths will a person take in her lifetime?

$$(3 \times 10^4) \times (3 \times 10^4) = 9 \times 10^8$$

The average American takes about 900,000,000 breaths in a lifetime.

Closing (2 minutes)

Summarize the lesson for the students.

- In general, close approximation of quantities will lead to more precise answers.
- We can multiply and divide numbers that are written in the form of a single-digit integer times a power of 10.

Exit Ticket (4 minutes)

Fluency Exercise (10 minutes)

Sprint: Practice the laws of exponents. Instruct students to write answers using positive exponents only. This exercise can be administered at any point during the lesson. Refer to the Sprints and Sprint Delivery Script sections in the Module Overview for directions to administer a Sprint.



Name _____

Date _____

Lesson 8: Estimating Quantities

Exit Ticket

Most English-speaking countries use the short-scale naming system, in which a trillion is expressed as 1,000,000,000,000. Some other countries use the long-scale naming system, in which a trillion is expressed as 1,000,000,000,000,000,000. Express each number as a single-digit integer times a power of ten. How many times greater is the long-scale naming system than the short-scale?

Exit Ticket Sample Solution

Most English-speaking countries use the short-scale naming system, in which a trillion is expressed as 1,000,000,000,000. Some other countries use the long-scale naming system, in which a trillion is expressed as 1,000,000,000,000,000,000,000,000. Express each number as a single-digit integer times a power of ten. How many times greater is the long-scale naming system than the short-scale?

$$1\,000\,000\,000\,000 = 10^{12}$$

$$1\,000\,000\,000\,000\,000\,000\,000 = 10^{21}$$

$$\frac{10^{21}}{10^{12}} = 10^9. \text{ The long-scale is about } 10^9 \text{ times greater than the short-scale.}$$

Problem Set Sample Solutions

Students practice estimating size of quantities and performing operations on numbers written in the form of a single-digit integer times a power of 10.

- The Atlantic Ocean region contains approximately 2×10^{16} gallons of water. Lake Ontario has approximately 8,000,000,000,000 gallons of water. How many Lake Ontarios would it take to fill the Atlantic Ocean region in terms of gallons of water?

$$8\,000\,000\,000\,000 = 8 \times 10^{12}$$

$$\begin{aligned} \frac{2 \times 10^{16}}{8 \times 10^{12}} &= \frac{2}{8} \times \frac{10^{16}}{10^{12}} \\ &= \frac{1}{4} \times 10^4 \\ &= 0.25 \times 10^4 \\ &= 2500 \end{aligned}$$

2,500 Lake Ontario's would be needed to fill the Atlantic Ocean region.

- U.S. national forests cover approximately 300,000 square miles. Conservationists want the total square footage of forests to be $300,000^2$ square miles. When Ivanna used her phone to do the calculation, her screen showed the following:



- What does the answer on her screen mean? Explain how you know.

The answer means 9×10^{10} . This is because:

$$\begin{aligned} (300\,000)^2 &= (3 \times 10^5)^2 \\ &= 3^2 \times (10^5)^2 \\ &= 9 \times 10^{10} \end{aligned}$$

- Given that the U.S. has approximately 4 million square miles of land, is this a reasonable goal for conservationists? Explain.

$4\,000\,000 = 4 \times 10^6$. It is unreasonable for conservationists to think the current square mileage of forests could increase that much because that number is greater than the number that represents the total number of square miles in the U.S., $9 \times 10^{10} > 4 \times 10^6$.



3. The average American is responsible for about 20,000 kilograms of carbon emission pollution each year. Express this number as a single-digit integer times a power of 10.

$$20\,000 = 2 \times 10^4$$

4. The United Kingdom is responsible for about 1×10^4 kilograms of carbon emission pollution each year. Which country is responsible for greater carbon emission pollution each year? By how much?

$$2 \times 10^4 > 1 \times 10^4$$

America is responsible for greater carbon emission pollution each year. America produces twice the amount of the U.K. pollution.