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A Synthesis of Modeling with Equations and Functions

Topics A and B (assessment 2 days, return 4 days, remediation or further applications 5 days)

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[1]Each lesson is one day, and one day is considered a 45-minute period.

Algebra I • Module 5

A Synthesis of Modeling with Equations and Functions

OVERVIEW

In Grade 8, students used functions for the first time to construct a function that models a linear relationship between two quantities (**8.F.B.4**) and to describe qualitatively the functional relationship between two quantities by analyzing a graph (**8.F.B.5**). In the first four modules of Algebra I, students learn to create and apply linear, quadratic, and exponential functions in addition to square and cube root functions (**F-IF.C.7**). In Module 5, they synthesize what they have learned during the year by selecting the correct function type in a series of modeling problems without the benefit of a module or lesson title that includes function type to guide them in their choices. This supports the CCLS requirement that student's use the modeling cycle, in the beginning of which they must formulate a strategy. Skills and knowledge from the previous modules support the requirements of this module, including writing, rewriting, comparing, and graphing functions (**F-IF.C.7**, **F-IF.C.8**, **F-IF.C.9**) and interpretation of the parameters of an equation (**F-LE.B.5**). Students also draw on their study of statistics in Module 2, using graphs and functions to model a context presented with data and tables of values (**S-ID.B.6**). In this module, we use the modeling cycle (see page 72 of the CCLS) as the organizing structure rather than function type.
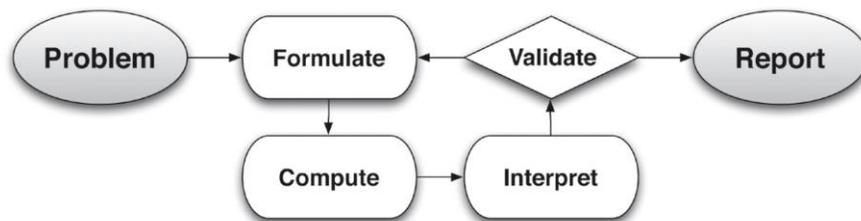
Topic A focuses on the skills inherent in the modeling process: representing graphs, data sets, or verbal descriptions using explicit expressions (**F-BF.A.1a**). Information is presented in graphic form in Lesson 1, as data in Lesson 2, and as a verbal description of a contextual situation in Lesson 3. Students recognize the function type associated with the problem (**F-LE.A.1b**, **F-LE.A.1c**) and match to or create 1- and 2-variable equations (**A-CED.A.1**, **A-CED.A.2**) to model a context presented graphically, as a data set, or as a description (**F-LE.A.2**). Function types include linear, quadratic, exponential, square root, cube root, absolute value, and other piecewise functions. Students interpret features of a graph in order to write an equation that can be used to model it and the function (**F-IF.B.4**, **F-BF.A.1**) and relate the domain to both representations (**F-IF.B.5**). This topic focuses on the skills needed to complete the modeling cycle and sometimes uses purely mathematical models, sometimes real-world contexts.

Tables, graphs, and equations all represent models. We use terms such as *symbolic* or *analytic* to refer specifically to the equation form of a function model; *descriptive model* refers to a model that seeks to describe or summarize phenomena, such as a graph. In Topic B, students expand on their work in Topic A to complete the modeling cycle for a real-world contextual problem presented as a graph, a data set, or a verbal description. For each, they *formulate* a function model, perform *computations* related to solving the problem, *interpret* the problem and the model, and then *validate* through iterations of revising their models as needed, and *report* their results.

Students choose and define the quantities of the problem (**N-Q.A.2**) and the appropriate level of precision for the context (**N-Q.A.3**). They create 1- and 2-variable equations (**A-CED.A.1**, **A-CED.A.2**) to model the context when presented as a graph, as data, and as a verbal description. They can distinguish among situations that represent a linear (**F-LE.A.1b**), quadratic, or exponential (**F-LE.A.1c**) relationship. For data, they look for first differences to be constant for linear relationships, second differences to be constant for quadratic relationships, and a common ratio for exponential relationships. When there are clear patterns in the data, students will recognize when the pattern represents a linear (arithmetic) or exponential (geometric) sequence (**F-BF.A.1a**, **F-LE.A.2**). For graphic presentations, students interpret the key features of the graph, and for both data sets and verbal descriptions, students sketch a graph to show the key features (**F-IF.B.4**). They calculate and interpret the average rate of change over an interval, estimating when using the graph (**F-IF.B.6**), and relate the domain of the function to its graph and to its context (**F-IF.B.5**).

Focus Standards

Reason quantitatively and use units to solve problems.

N-Q.A.2[2]   Define appropriate quantities for the purpose of descriptive modeling.★

N-Q.A.3   Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

Create equations that describe numbers or relationships.

A-CED.A.1[3]   Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.★

A-CED.A.2   Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

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[2]This standard will be assessed in Algebra I by ensuring that some modeling tasks (involving Algebra I content or securely-held content from Grades 6–8) require the student to create a quantity of interest in the situation being described.
[3]In Algebra I, tasks are limited to linear, quadratic, or exponential equations with integer exponents.

Interpret functions that arise in applications in terms of the context.

F-IF.B.4[4]   For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

F-IF.B.5   Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

F-IF.B.6[5]   Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Build a function that models a relationship between two quantities.

F-BF.A.1[6]   Write a function that describes a relationship between two quantities.★

a.   Determine an explicit expression, a recursive process, or steps for calculation from a context.

Construct and compare linear, quadratic, and exponential models and solve problems.

F-LE.A.1   Distinguish between situations that can be modeled with linear functions and with exponential functions.★

b.   Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c.   Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.A.2[7]   Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★

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[4]Tasks have a real-world context. In Algebra I, tasks are limited to linear, quadratic, square root, cube root, piecewise-defined (including step and absolute value functions), and exponential functions with domains in the integers.

[5]Tasks have a real-world context. In Algebra I, tasks are limited to linear, quadratic, square root, cube root, piecewise-defined (including step functions and absolute value functions), and exponential functions with domains in the integers.

[6]Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, and exponential functions with domains in the integers.

[7]In Algebra I, tasks are limited to constructing linear and exponential functions in simple context (not multi-step).

Foundational Standards

Use functions to model relationships between quantities.

8.F.B.4   Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.B.5   Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Analyze functions using different representations.

F-IF.C.7   Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

   a.   Graph linear and quadratic functions and show intercepts, maxima, and minima.

   b.   Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-IF.C.8   Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

   a.   Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F-IF.C.9[8]   Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Interpret expressions for functions in terms of the situation they model.

F-LE.B.5[9]   Interpret the parameters in a linear or exponential function in terms of a context.★

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[8]In Algebra I, tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.
[9]Tasks have a real-world context. In Algebra I, exponential functions are limited to those with domains in the integers.

Summarize, represent, and interpret data on two categorical and quantitative variables.

S-ID.B.6[10]   Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

    a.   Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models*.

    b.   Informally assess the fit of a function by plotting and analyzing residuals.

Focus Standards for Mathematical Practice

MP.1   **Make sense of problems and persevere in solving them.** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. In this module, students make sense of the problem by analyzing the critical components of the problem, presented as a verbal description, a data set, or a graph and persevere in writing the appropriate function that describes the relationship between two quantities. Then, they interpret the function in the context.

MP.2   **Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. This module alternates between algebraic manipulation of expressions and equations and interpreting the quantities in the relationship in terms of the context. In Topic A, students develop fluency in recognizing and identifying key features of the three primary function types studied in Algebra I, as well as manipulating expressions to highlight those features. Topic B builds on these skills so that when students are given a verbal description of a situation that can be described by a function, they decontextualize it and apply the skills they learned in Topic A in order to further analyze the situation. Then, they contextualize their work so they can compare, interpret, and make predictions and claims. In the assessment, students are frequently asked to explain their solutions so that teachers have a clear understanding of the reasoning behind their results.

MP.4   **Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In this module, students create a function from a contextual situation described verbally, create a graph of their function, interpret key features of both the function and the graph in the terms of the context, and answer questions related to the function and its graph. They also create a function from a data set based on a contextual situation. In Topic B, students use the full modeling cycle with functions presented mathematically or in a context, including linear, quadratic, and exponential. They explain their mathematical thinking in writing and using appropriate tools, such as graph paper, graphing calculators, or computer software.

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[10]Tasks have a real-world context. In Algebra I, exponential functions are limited to those with domains in the integers.

MP.5   **Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Throughout the entire module students must decide whether or not to use a tool to help find solutions. They must graph functions that are sometimes difficult to sketch (e.g., cube root and square root) and sometimes are required to perform procedures that can be tedious, and sometimes detract from the mathematical thinking, when performed without technology (e.g., completing the square with non-integer coefficients). In these cases, students must decide whether to use a tool to help with the calculation or graph so they can better analyze the model. Students should have access to a graphing calculator for use on the module assessment.

MP.6   **Attend to precision.** Mathematically proficient students try to communicate precisely to others. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. When calculating and reporting quantities in all topics of this module, students must choose the appropriate units and use the appropriate level of precision based on the information as it is presented. When graphing, they must select an appropriate scale.

Terminology

Note: This module is a synthesis of all concepts learned in Algebra I.

New or Recently Introduced Terms

- **Analytic Model** (A model that seeks to explain data based on deeper theoretical ideas. For example, by using an algebraic equation. This is sometimes referred to as a symbolic model.)
- **Descriptive Model** (A model that seeks to describe phenomena or summarize them in a compact form. For example, by using a graph.)

Familiar Terms and Symbols[11]

- Analytical Model
- Arithmetic Sequence
- Average Rate of Change
- Cube Root Function
- End Behavior
- Exponential Function

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[11]These are terms and symbols students have seen previously.

- First Differences
- Function
- Geometric Sequence
- Linear Function
- Parameter
- Parent Function
- Piecewise Defined Function
- Quadratic Function
- Range
- Recursive Process
- Square Root Function
- Second Differences

Suggested Tools and Representations

- Scientific Calculator
- Graphing Calculator
- Geometer's Sketch Pad
- GeoGebra

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
End-of-Module Assessment Task	After Topic B	Constructed response with rubric	N-Q.A.2, N-Q.A.3, A-CED.A.1, A-CED.A.2, F-IF.B.4, F-IF.B.5, F-IF.B.6, F-BF.A.1, F-LE.A.1, F-LE.A.2