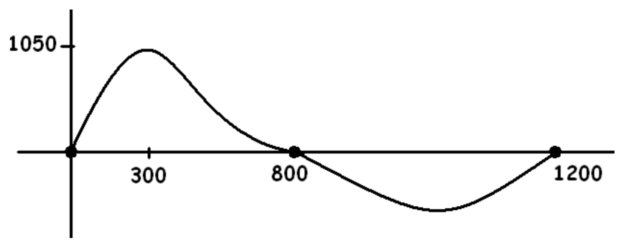
Name ______________________________ Date ______________

1. Geographers sit at a café discussing their field work site, which is a hill and a neighboring riverbed. The hill is approximately $1{,}050$ feet high, 800 feet wide, with peak about 300 feet east of the western base of the hill. The river is about 400 feet wide. They know the river is shallow, no more than about 20 feet deep.

   They make the following crude sketch on a napkin, placing the profile of the hill and riverbed on a coordinate system with the horizontal axis representing ground level.



   The geographers do not have any computing tools with them at the café, so they decide to use pen and paper to compute a cubic polynomial that approximates this profile of the hill and riverbed.

   a. Using only a pencil and paper, write a cubic polynomial function H that could represent the curve shown (here, x represents the distance, in feet, along the horizontal axis from the western base of the hill, and $H(x)$ is the height, in feet, of the land at that distance from the western base). Be sure that your formula satisfies $H(300) = 1050$.

b. For the sake of convenience, the geographers make the assumption that the deepest point of the river is halfway across the river (recall that the river is no more than 20 feet deep). Under this assumption, would a cubic polynomial provide a suitable model for this hill and riverbed? Explain.

2. Luke notices that by taking any three consecutive integers, multiplying them together, and adding the middle number to the result, the answer always seems to be the middle number cubed.

For example:

$$3 \times 4 \times 5 + 4 = 64 = 4^3$$
$$4 \times 5 \times 6 + 5 = 125 = 5^3$$
$$9 \times 10 \times 1\,1 + 10 = 1000 = 10^3$$

a. To prove his observation, Luke writes $(n+1)(n+2)(n+3)+(n+2)$. What answer is he hoping to show this expression equals?

b. Lulu, upon hearing of Luke's observation, writes her own version with n as the middle number. What does her formula look like?

c. Use Lulu's expression to prove that adding the middle number to the product of any three consecutive numbers is sure to equal that middle number cubed.

3. A cookie company packages its cookies in rectangular prism boxes designed with square bases that have both a length and width of 4 inches less than the height of the box.

a. Write a polynomial that represents the volume of a box with height x inches.

b. Find the dimensions of the box if its volume is 128 cubic inches.

c. After solving this problem, Juan was very clever and invented the following strange question:

A building, in the shape of a rectangular prism with a square base, has on its top a radio tower. The building is 25 *times as tall as the tower, and the side-length of the base of the building is* 100 *feet less than the height of the building. If the building has a volume of* 2 *million cubic feet, how tall is the tower?*

Solve Juan's problem.

A Progression Toward Mastery					
Assessment Task Item		**STEP 1** Missing or incorrect answer and little evidence of reasoning or application of mathematics to solve the problem.	**STEP 2** Missing or incorrect answer but evidence of some reasoning or application of mathematics to solve the problem.	**STEP 3** A correct answer with some evidence of reasoning or application of mathematics to solve the problem OR an incorrect answer with substantial evidence of solid reasoning or application of mathematics to solve the problem.	**STEP 4** A correct answer supported by substantial evidence of solid reasoning or application of mathematics to solve the problem.
1	**a** N-Q.A.2 A-APR.B.2 A-APR.B.3 F-IF.C.7c	Student identifies zeros on the graph.	Student uses zeros to write a factored cubic polynomial for $H(x)$ without a leading coefficient.	Student uses given condition $H(300) = 1050$ to find a-value (leading coefficient).	Student writes a complete cubic model for $H(x)$ in factored form with correct a-value (leading coefficient).
	b N-Q.A.2 A-APR.B.2 A-APR.B.3 F-IF.C.7c	Student finds the mid-point of the river.	Student evaluates $H(x)$ using the midpoint. The exact answer is not needed, only an approximation.	Student determines if a cubic model is suitable for this hill and riverbed.	Student justifies the answer using $H(midpoint)$ in the explanation.
2	**a** A-SSE.A.2 A-APR.C.4	Student does not indicate any expression involving n raised to an exponent of 3.	Student uses a base involving n being raised to an exponent of 3 in the answer but does not choose a base of $(n + 2)$.	Student writes $(n + 2)^3$ without including parentheses to indicate all of $(n + 2)$ is being cubed (i.e., $n + 2^3$). OR Student makes another error that shows general understanding but is technically incorrect.	Student writes the correct answer, $(n + 2)^3$.

	b–c **A-SSE.A.2** **A-APR.C.4**	Student does not answer parts (b)–(c). OR Student provides incorrect or incomplete answers.	Student answers part (b) incorrectly but uses correct algebra in showing equivalence to n^3. OR Student answers part (b) correctly but makes major errors or is unable to show its equivalence to n^3.	Student answers part (b) correctly as $(n-1)(n)(n+1)+n=n^3$ but makes minor errors in showing equivalence to n^3.	Student answers correctly as $(n-1)(n)(n+1)+n=n^3$ and correctly multiplies the left side and then combines like terms to show equivalence to n^3.
3	**a–d** **N-Q.A.2** **A-SSE.A.2** **A-APR.B.2** **A-APR.B.3** **A-REI.A.1** **A-REI.B.4b**	Student determines an expression for $V(x)$.	Student sets $V(x)$ equal to the given volume.	Student solves the equation understanding that only real values are possible solutions for the dimensions of a box.	Student states the three dimensions of the box with proper units.
	c **N-Q.A.2** **A-SSE.A.2** **A-APR.B.2** **A-APR.B.3** **A-REI.A.1** **A-REI.B.4b**	Student determines an expression for $V(h)$ and sets it equal to the given volume but does not solve the equation.	Student writes the equation as a polynomial equation but is unable to find any solutions to the equation.	Student finds one solution to the equation but is unable to use it to factor the polynomial expression to find the other potential solutions. OR Student recognizes the simplified equation as the same as the equation in the previous part and states the solution from the previous part immediately (but as the real number 8, not as a height measurement 8 feet).	Student finds the solutions to the equation, determines which solution is valid, and states the correct answer as the height measurement 8 feet. OR Student recognizes the simplified equation as the same as the equation in the previous part and states the solution from the previous part immediately using the correct units (i.e., 8 feet).

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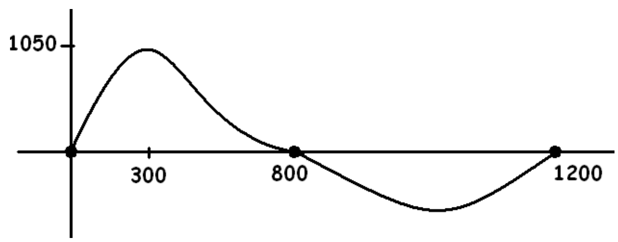

   The geographers do not have any computing tools with them at the café, so they decide to use pen and paper to compute a cubic polynomial that approximates this profile of the hill and riverbed.

   a. Using only a pencil and paper, write a cubic polynomial function H that could represent the curve shown (here, x represents the distance, in feet, along the horizontal axis from the western base of the hill, and $H(x)$ is the height, in feet, of the land at that distance from the western base). Be sure that your formula satisfies $H(300) = 1050$.

We have $H(x) = cx(x-800)(x-1200)$. For $H(300)$ equal 1050, we need

$$1050 = c(300)(-500)(-900)$$

$$5 \times 21 \times 10 = c \times 3 \times 5 \times 9 \times 10^6$$

$$7 = c \times 9 \times 10^5$$

$$c = \frac{7}{9 \times 10^5}$$

So $H(x) = \frac{7}{9 \times 10^5} x(x-800)(x-1200)$

b. For the sake of convenience, the geographers make the assumption that the deepest point of the river is halfway across the river (recall that the river is no more than 20 feet deep). Under this assumption, would a cubic polynomial provide a suitable model for this hill and riverbed? Explain.

Halfway across the river is $x = 1000$, and
$H(1000) = \frac{7}{9 \times 10^5}(1000)(200)(-200) = -\frac{7 \times 2 \times 2 \times 100}{9} = -\frac{2800}{9}$

Notice that $\frac{2800}{9} > \frac{2700}{9} = 300$, so this model says that the river is over 300 ft. deep!
This is not a good model for a shallow river, no more than 20 ft. deep.

2. Luke notices that by taking any three consecutive integers, multiplying them together, and adding the middle number to the result, the answer always seems to be the middle number cubed.

For example: $3 \times 4 \times 5 + 4 = 64 = 4^3$
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a. To prove his observation, Luke writes $(n+1)(n+2)(n+3)+(n+2)$. What answer is he hoping to show this expression equals?

$(n+2)^3$

b. Lulu, upon hearing of Luke's observation, writes her own version with n as the middle number. What does her formula look like?

$(n-1)n(n+1) + n = n^3$

c. Use Lulu's expression to prove that adding the middle number to the product of any three consecutive numbers is sure to equal that middle number cubed.

We need to show $(n-1)n(n+1)$ equals n^3.

Now, $(n-1)n(n+1)+n = n(n-1)(n+1)+n$

$= n(n^2-1)+n$

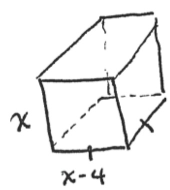
$= n^3-n+n$

$= n^3$

It does!

3. A cookie company packages its cookies in rectangular prism boxes designed with square bases that have both a length and width of 4 inches less than the height of the box.

a. Write a polynomial that represents the volume of a box with height x inches.



$V = x(x-4)^2$

b. Find the dimensions of the box if its volume is 128 cubic inches.

$128 = x(x-4)^2$

$0 = (x^2-8x+16)x - 128$

$0 = x^3-8x^2+16x-128$

$0 = (x^2+16)(x-8)$

So either $x-8=0$ giving $x=8$

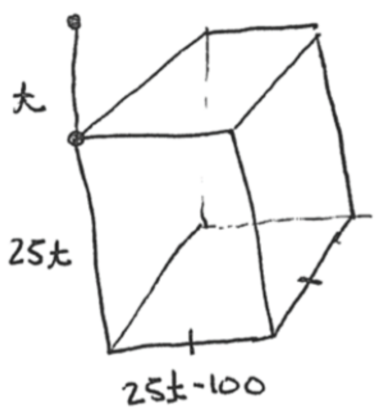
or $x^2+16=0$. But this second equation has no real solutions.

So $x=8$ is the only solution, and the dimensions of the box are $4" \times 4" \times 8"$.

c. After solving this problem, Juan was very clever and invented the following strange question:

A building, in the shape of a rectangular prism with a square base, has on its top a radio tower. The building is 25 *times as tall as the tower, and the side-length of the base of the building is* 100 *feet less than the height of the building. If the building has a volume of* 2 *million cubic feet, how tall is the tower?*

Solve Juan's problem.



$V = 25t(25t-100)^2 = 2{,}000{,}000$

$25t(25(t-4))^2 = 2{,}000{,}000$

$25^3 t(t-4)^2 = 2{,}000{,}000$

$t(t-4)^2 = 128$

Since this is the same equation as part b, we know the solution is $t = 8$.

The tower is 8 ft. tall.