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Grade 8 • Module 4

Linear Equations

OVERVIEW

In Module 4, students extend what they already know about unit rates and proportional relationships (**6.RP.A.2**, **7.RP.A.2**) to linear equations and their graphs. Students understand the connections between proportional relationships, lines, and linear equations in this module (**8.EE.B.5**, **8.EE.B.6**). Also, students learn to apply the skills they acquired in Grades 6 and 7 with respect to symbolic notation and properties of equality (**6.EE.A.2**, **7.EE.A.1**, **7.EE.B.4**) to transcribe and solve equations in one variable and then in two variables.

In Topic A, students begin by transcribing written statements using symbolic notation. Then, students write linear and nonlinear expressions leading to linear equations, which are solved using properties of equality (**8.EE.C.7b**). Students learn that not every linear equation has a solution. In doing so, students learn how to transform given equations into simpler forms until an equivalent equation results in a unique solution, no solution, or infinitely many solutions (**8.EE.C.7a**). Throughout Topic A, students must write and solve linear equations in real-world and mathematical situations.

In Topic B, students work with constant speed, a concept learned in Grade 6 (**6.RP.A.3**), but this time with proportional relationships related to average speed and constant speed. These relationships are expressed as linear equations in two variables. Students find solutions to linear equations in two variables, organize them in a table, and plot the solutions on a coordinate plane (**8.EE.C.8a**). It is in Topic B that students begin to investigate the shape of a graph of a linear equation. Students predict that the graph of a linear equation is a line and select points on and off the line to verify their claim. Also in this topic is the standard form of a linear equation, $ax + by = c$, and when $a \neq 0$ and $b \neq 0$, a non-vertical line is produced. Further, when $a = 0$ or $b = 0$, then a vertical or horizontal line is produced.

In Topic C, students know that the slope of a line describes the rate of change of a line. Students first encounter slope by interpreting the unit rate of a graph (**8.EE.B.5**). In general, students learn that slope can be determined using any two distinct points on a line by relying on their understanding of properties of similar triangles from Module 3 (**8.EE.B.6**). Students verify this fact by checking the slope using several pairs of points and comparing their answers. In this topic, students derive $y = mx$ and $y = mx + b$ for linear equations by examining similar triangles. Students generate graphs of linear equations in two variables first by completing a table of solutions and then by using information about slope and y-intercept. Once students are sure that every linear equation graphs as a line and that every line is the graph of a linear equation, students graph equations using information about x- and y-intercepts. Next, students learn some basic facts about lines and equations, such as why two lines with the same slope and a common point are the same line, how to write equations of lines given slope and a point, and how to write an equation given two points. With the concepts of slope and lines firmly in place, students compare two different proportional relationships represented by graphs, tables, equations, or descriptions. Finally, students learn that multiple forms of an equation can define the same line.

Simultaneous equations and their solutions are the focus of Topic D. Students begin by comparing the constant speed of two individuals to determine which has greater speed (**8.EE.C.8c**). Students graph simultaneous linear equations to find the point of intersection and then verify that the point of intersection is in fact a solution to each equation in the system (**8.EE.C.8a**). To motivate the need to solve systems algebraically, students graph systems of linear equations whose solutions do not have integer coordinates. Students learn to solve systems of linear equations by substitution and elimination (**8.EE.C.8b**). Students understand that a system can have a unique solution, no solution, or infinitely many solutions, as they did with linear equations in one variable. Finally, students apply their knowledge of systems to solve problems in real-world contexts, including converting temperatures from Celsius to Fahrenheit.

Optional Topic E is an application of systems of linear equations (**8.EE.C.8b**). Specifically, this system generates Pythagorean triples. First, students learn that a Pythagorean triple can be obtained by multiplying any known triple by a positive integer (**8.G.B.7**). Then, students are shown the Babylonian method for finding a triple that requires the understanding and use of a system of linear equations.

Focus Standards

Understand the connections between proportional relationships, lines, and linear equations.

8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example,* compare *a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.C.7 Solve linear equations in one variable.

a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,* $3x + 2y = 5$ *and* $3x + 2y = 6$ *have no solution because* $3x + 2y$ *cannot simultaneously be* 5 *and* 6.

c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Foundational Standards

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.A.2 Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. For *example, "This recipe has a ratio of* 3 *cups of flour to* 4 *cups of sugar, so there is* $3/4$ *cup of flour for each cup of sugar." "We paid* $\$75$ *for* 15 *hamburgers, which is a rate of* $\$5$ *per hamburger."*[2]

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took* 7 *hours to mow* 4 *lawns, then at that rate, how many lawns could be mowed in* 35 *hours? At what rate were lawns being mowed?*

Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.A.2 Write, read, and evaluate expressions in which letters stand for numbers.

a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract* y *from* 5*" as* $5 - y$.

b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression* $2(8 + 7)$ *as a product of two factors; view* $(8 + 7)$ *as both a single entity and a sum of two terms.*

[2] Expectations for unit rates in this grade are limited to non-complex fractions.

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with side length $s = 1/2$.*

Analyze proportional relationships and use them to solve real-world and mathematical problems.

7.RP.A.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.*

d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

Use properties of operations to generate equivalent expressions.

7.EE.A.1 Apply properties of operations as **strategies** to add, subtract, factor, and expand linear expressions with rational coefficients.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

7.EE.B.4 Use variables to **represent** quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p,q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students analyze given constraints to make conjectures about the form and meaning of a solution to a given situation in one-variable and two-variable linear equations, as well as in simultaneous linear equations. Students are systematically guided to understand the meaning of a linear equation in one variable, the natural occurrence of linear equations in two variables with respect to proportional relationships, and the natural emergence of a system of two linear equations when looking at related, continuous proportional relationships.

MP.2 **Reason abstractly and quantitatively.** Students decontextualize and contextualize throughout the module as they represent situations symbolically and make sense of solutions within a context. Students use facts learned about rational numbers in previous grade levels to solve linear equations and systems of linear equations.

MP.3 **Construct viable arguments and critique the reasoning of others.** Students use assumptions, definitions, and previously established facts throughout the module as they solve linear equations. Students make conjectures about the graph of a linear equation being a line and then proceed to prove this claim. While solving linear equations, they learn that they must first assume that a solution exists and then proceed to solve the equation using properties of equality based on the assumption. Once a solution is found, students justify that it is in fact a solution to the given equation, thereby verifying their initial assumption. This process is repeated for systems of linear equations.

MP.4 **Model with mathematics.** Throughout the module, students represent real-world situations symbolically. Students identify important quantities from a context and represent the relationship in the form of an equation, a table, and a graph. Students analyze the various representations and draw conclusions and/or make predictions. Once a solution or prediction has been made, students reflect on whether the solution makes sense in the context presented. One example of this is when students determine how many buses are needed for a field trip. Students must interpret their fractional solution and make sense of it as it applies to the real world.

MP.7 **Look for and make use of structure.** Students use the structure of an equation to make sense of the information in the equation. For example, students write equations that represent the constant rate of motion for a person walking. In doing so, they interpret an equation such as $y = \frac{3}{5}x$ as the total distance a person walks, y, in x amount of time, at a rate of $\frac{3}{5}$. Students look for patterns or structure in tables and show that a rate is constant.

Terminology

New or Recently Introduced Terms

- **Average Speed** (Let a time interval of t hours be given. Suppose that an object travels a total distance of d miles during this time interval. The *average speed of the object in the given time interval* is $\frac{d}{t}$ miles per hour.
- **Constant Speed** (For any positive real number v, an object travels at a *constant speed of v mph* over a fixed time interval if the average speed is always equal to v mph for any smaller time interval of the given time interval.)
- **Horizontal Line** (In a Cartesian plane, a *horizontal line* is either the x-axis or any other line parallel to the x-axis. For example, the graph of the equation $y = -5$ is a horizontal line.)
- **Linear Equation (description)** (A *linear equation* is an equation in which both expressions are linear expressions.)
- **Point-Slope Equation of a Line** (The *point-slope equation of a non-vertical line* in the Cartesian plane that passes through point (x_1, y_1) and has slope m is
$$y - y_1 = m(x - x_1).$$
It can be shown that every non-vertical line is the graph of its point-slope equation, and that every graph of a point-slope equation is a line.)
- **Slope of a Line in a Cartesian Plane** (The *slope* of a non-vertical line in a Cartesian plane that passes through two different points is the number given by the change in y-coordinates divided by the corresponding change in the x-coordinates. For two points (x_1, y_1) and (x_2, y_2) on the line where $x_1 \neq x_2$, the slope of the line m can be computed by the formula
$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$
The slope of a vertical line is not defined. The definition of slope is well-defined after one uses similar triangles to show that expression $\frac{y_2 - y_1}{x_2 - x_1}$ is always the same number for any two distinct points (x_1, y_1) and (x_2, y_2) on the line.)
- **Slope-Intercept Equation of a Line** (The *slope-intercept equation of a non-vertical line* in the Cartesian plane with slope m and y-intercept b is
$$y = mx + b.$$
It can be shown that every non-vertical line is the graph of its slope-intercept equation, and that every graph of a slope-intercept equation is a line.)
- **Solution to a System of Linear Equations (description)** (A *solution to a system of two linear equations in two variables* is an ordered pair of numbers that is a solution to both equations. For example, the solution to the system of linear equations $\begin{cases} x + y = 6 \\ x - y = 4 \end{cases}$ is the ordered pair $(5, 1)$ because substituting 5 in for x and 1 in for y results in two true equations: $5 + 1 = 6$ and $5 - 1 = 4$.)

- **Standard Form of a Linear Equation** (A linear equation in two variables x and y is in *standard form* if it is of the form

$$ax + by = c$$

for real numbers a, b, and c, where a and b are both not zero. The numbers a, b, and c are called *constants*.)

- **System of Linear Equations** (A *system of linear equations* is a set of two or more linear equations. For example, $\begin{cases} x + y = 15 \\ 3x - 7y = -2 \end{cases}$ is a system of linear equations.)
- **Vertical Line** (In a Cartesian plane, a *vertical line* is either the y-axis or any other line parallel to the y-axis. For example, the graph of the equation $x = 3$ is a vertical line.)
- **X-Intercept** (An x-*intercept* of a graph is the x-coordinate of a point where the graph intersects the x-axis. An x-*intercept point* is the coordinate point where the graph intersects the x-axis.

  The x-intercept of a graph of a linear equation can be found by setting $y = 0$ in the equation. Many times the term "x-intercept point" is shortened to just "x-intercept" if it is clear from the context that the term is referring to a point and not a number.
- **Y-Intercept** (A y-*intercept* of a graph is the y-coordinate of a point where the graph intersects the y-axis. A y-*intercept point* is the coordinate point where the graph intersects the y-axis.

  The y-intercept of a graph of a linear equation can be found by setting $x = 0$ in the equation. Many times the term "y-intercept point" is shortened to just "y-intercept" if it is clear from the context that the term is referring to a point and not a number.)

Familiar Terms and Symbols[3]

- Coefficient
- Equation
- Like terms
- Linear Expression
- Solution
- Term
- Unit rate
- Variable

Suggested Tools and Representations

- Scientific calculator
- Online graphing calculator (e.g., https://www.desmos.com/calculator)
- Graph paper
- Straightedge

[3]These are terms and symbols students have seen previously.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	8.EE.C.7, 8.EE.B.5
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	8.EE.B.5, 8.EE.B.6, 8.EE.C.7, 8.EE.C.8