Lesson 9: Sequencing Rotations

Student Outcomes

- Students learn that sequences of rotations preserve lengths of segments as well as degrees of measures of angles.
- Students describe a sequence of rigid motions that would map a triangle back to its original position after being rotated around two different centers.

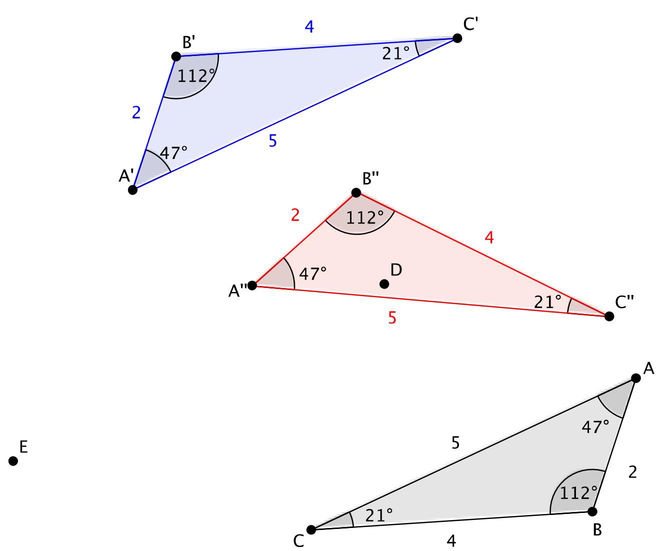
Classwork

Exploratory Challenge (35 minutes)

> *Scaffolding:*
> It may be beneficial to check student work periodically throughout the Exploratory Challenge. As necessary, provide teacher modeling.

Exploratory Challenge

1.



a. **Rotate $\triangle ABC$ d degrees around center D. Label the rotated image as $\triangle A'B'C'$.**

Sample student work is shown in blue.

b. **Rotate $\triangle A'B'C'$ d degrees around center E. Label the rotated image as $\triangle A''B''C''$.**

Sample student work is shown in red.

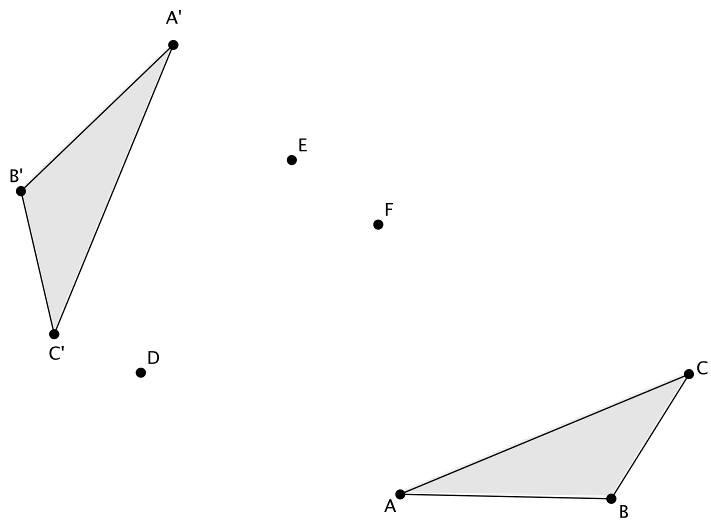
c. **Measure and label the angles and side lengths of $\triangle ABC$. How do they compare with the images $\triangle A'B'C'$ and $\triangle A''B''C''$?**

Measures of corresponding sides and measures of corresponding angles of three triangles are equal.

d. **How can you explain what you observed in part (c)? What statement can you make about properties of sequences of rotations as they relate to a single rotation?**

We already knew that a single rotation would preserve the lengths of segments and degrees of angles. Performing one rotation after the other does not change the lengths of segments or degrees of angles. That means that sequences of rotations enjoy the same properties as a single rotation.

2.



a. **Rotate $\triangle ABC$ d degrees around center D, and then rotate again d degrees around center E. Label the image as $\triangle A'B'C'$ after you have completed both rotations.**

Possible student solution is shown in diagram as $\triangle A'B'C'$.

b. **Can a single rotation around center D map $\triangle A'B'C'$ onto $\triangle ABC$?**

No, a single rotation around center D will not map $\triangle A'B'C'$ onto $\triangle ABC$.

c. **Can a single rotation around center E map $\triangle A'B'C'$ onto $\triangle ABC$?**

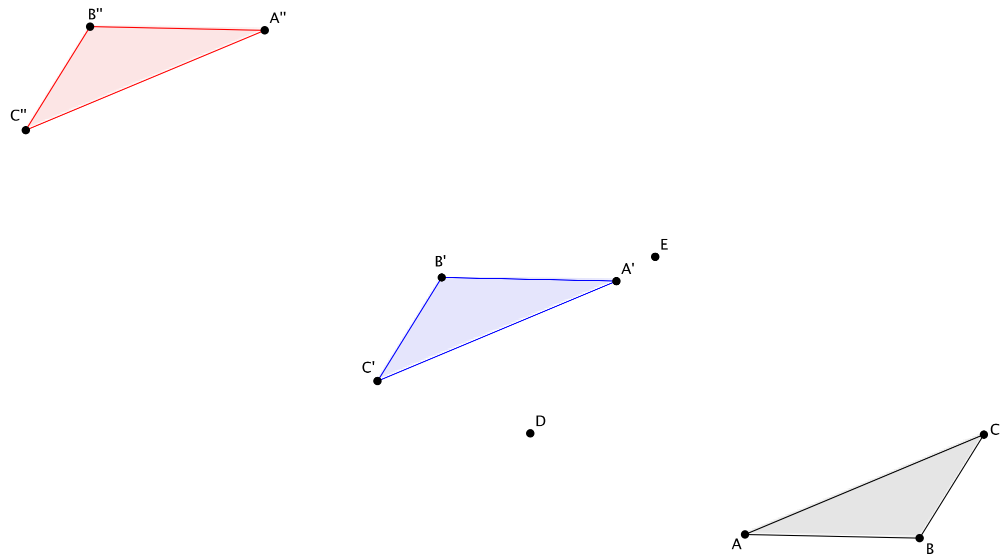
No, a single rotation around center E will not map $\triangle A'B'C'$ onto $\triangle ABC$.

d. **Can you find a center that would map $\triangle A'B'C'$ onto $\triangle ABC$ in one rotation? If so, label the center F.**

Yes, a d-degree rotation around center F will map $\triangle A'B'C'$ onto $\triangle ABC$.

Note: Students can only find the center F through trial and error at this point. Finding the center of rotation for two congruent figures is a skill that will be formalized in high school Geometry.

3.



a. Rotate $\triangle ABC$ $90°$ (counterclockwise) around center D, and then rotate the image another $90°$ (counterclockwise) around center E. Label the image $\triangle A'B'C'$.

Sample student work is shown in blue.

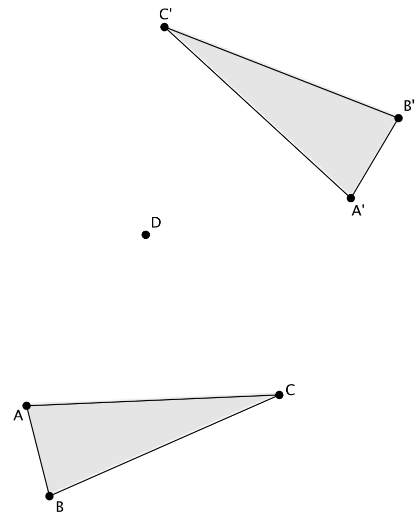
b. Rotate $\triangle ABC$ $90°$ (counterclockwise) around center E, and then rotate the image another $90°$ (counterclockwise) around center D. Label the image $\triangle A''B''C''$.

Sample student work is shown in red.

c. What do you notice about the locations of $\triangle A'B'C'$ and $\triangle A''B''C''$? Does the order in which you rotate a figure around different centers have an impact on the final location of the figure's image?

The triangles are in two different locations. Yes, the order in which we rotate a figure around two different centers must matter because the triangles are not in the same location after rotating around center D and then center E compared to rotating around center E and then center D.

4.



a. Rotate $\triangle ABC$ $90°$ (counterclockwise) around center D, and then rotate the image another $45°$ (counterclockwise) around center D. Label the image $\triangle A'B'C'$.

Rotated triangle is shown above.

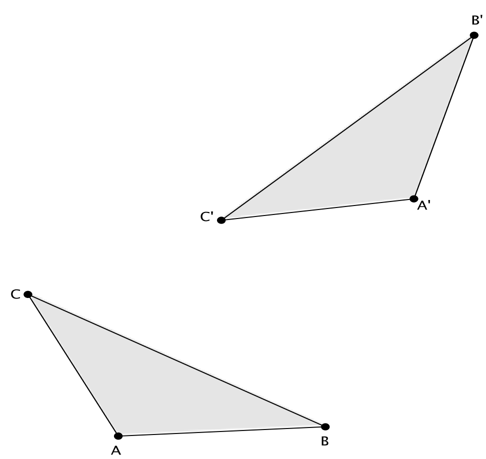
b. Rotate $\triangle ABC$ $45°$ (counterclockwise) around center D, and then rotate the image another $90°$ (counterclockwise) around center D. Label the image $\triangle A''B''C''$.

Rotated triangle is shown above.

c. What do you notice about the locations of $\triangle A'B'C'$ and $\triangle A''B''C''$? Does the order in which you rotate a figure around the same center have an impact on the final location of the figure's image?

The triangles are in the same location. This indicates that when a figure is rotated twice around the same center, it does not matter in which order you perform the rotations.

5. $\triangle ABC$ has been rotated around two different centers, and its image is $\triangle A'B'C'$. Describe a sequence of rigid motions that would map $\triangle ABC$ onto $\triangle A'B'C'$.



Translate $\triangle ABC$ along vector $\overrightarrow{CC'}$. Then, rotate $\triangle ABC$ around point C' until $\triangle ABC$ maps onto $\triangle A'B'C'$.

Closing (5 minutes)

Summarize, or have students summarize, the lesson.

- Sequences of rotations enjoy the same properties as single rotations. That is, a sequence of rotations preserves lengths of segments and degrees of measures of angles.
- The order in which a sequence of rotations around two different centers is performed matters. The order in which a sequence of rotations around the same center is performed does not matter.
- When a figure is rotated around two different centers, we can describe a sequence of rigid motions that would map the original figure onto the resulting image.

Lesson Summary

- **Sequences of rotations have the same properties as a single rotation:**
  - **A sequence of rotations preserves degrees of measures of angles.**
  - **A sequence of rotations preserves lengths of segments.**
- **The order in which a sequence of rotations around different centers is performed matters with respect to the final location of the image of the figure that is rotated.**
- **The order in which a sequence of rotations around the same center is performed does not matter. The image of the figure will be in the same location.**

Exit Ticket (5 minutes)

Name ____ Date ____________________

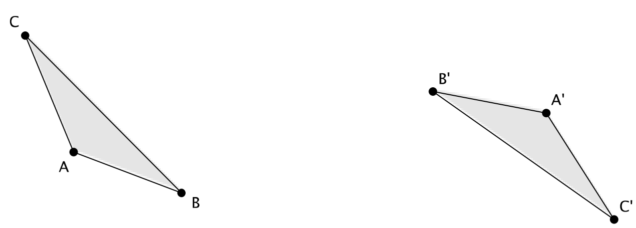
Lesson 9: Sequencing Rotations

Exit Ticket

1. Let $Rotation_1$ be the rotation of a figure d degrees around center O. Let $Rotation_2$ be the rotation of the same figure d degrees around center P. Does the $Rotation_1$ of the figure followed by the $Rotation_2$ equal a $Rotation_2$ of the figure followed by the $Rotation_1$? Draw a picture if necessary.

2. Angle ABC underwent a sequence of rotations. The original size of $\angle ABC$ Is $37°$. What was the size of the angle after the sequence of rotations? Explain.

3. Triangle ABC underwent a sequence of rotations around two different centers. Its image is $\triangle A'B'C'$. Describe a sequence of rigid motions that would map $\triangle ABC$ onto $\triangle A'B'C'$.



Exit Ticket Sample Solutions

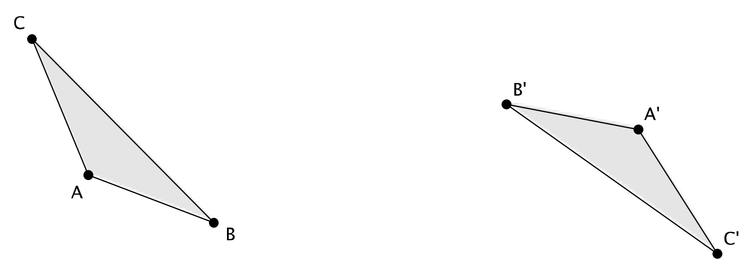
1. Let $Rotation_1$ be the rotation of a figure d degrees around center O. Let $Rotation_2$ be the rotation of the same figure d degrees around center P. Does the $Rotation_1$ of the figure followed by the $Rotation_2$ equal a $Rotation_2$ of the figure followed by the $Rotation_1$? Draw a picture if necessary.

   No. If the sequence of rotations were around the same center, then it would be true. However, when the sequence involves two different centers, the order in which they are performed matters because the images are not in the same location in the plane.

2. Angle ABC underwent a sequence of rotations. The original size of $\angle ABC$ is $37°$. What was the size of the angle after the sequence of rotations? Explain.

   Since sequences of rotations enjoy the same properties as a single rotation, then the measure of any image of $\angle ABC$ under any sequence of rotations remains $37°$. Rotations and sequences of rotations preserve the measure of degrees of angles.

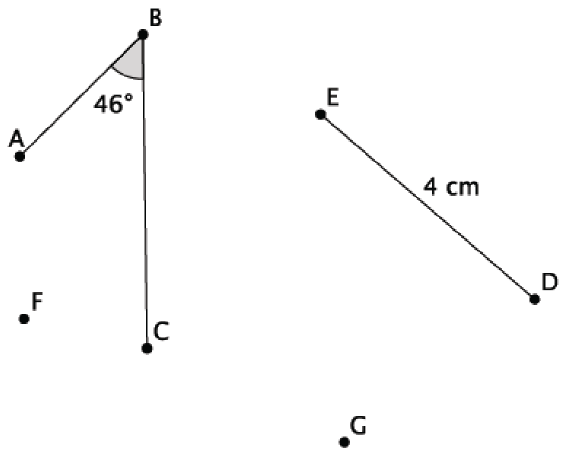
3. Triangle ABC underwent a sequence of rotations around two different centers. Its image is $\triangle A'B'C'$. Describe a sequence of rigid motions that would map $\triangle ABC$ onto $\triangle A'B'C'$.



   Translate $\triangle ABC$ along vector $\overrightarrow{BB'}$. Then, rotate $\triangle ABC$ d degrees around point B' until $\triangle ABC$ maps onto $\triangle A'B'C'$.

Problem Set Sample Solutions

1. Refer to the figure below.



a. Rotate $\angle ABC$ and segment DE d degrees around center F and then d degrees around center G. Label the final location of the images as $\angle A'B'C'$ and segment $D'E'$.

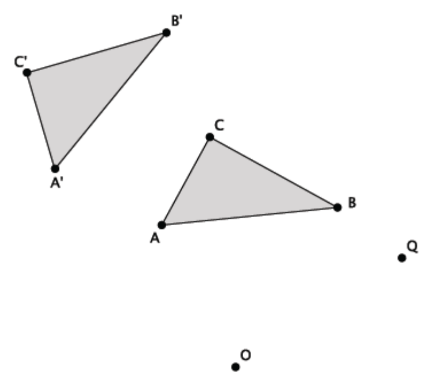
b. What is the size of $\angle ABC$, and how does it compare to the size of $\angle A'B'C'$? Explain.

The measure of $\angle ABC$. Is $46°$. The measure of $\angle A'B'C'$ is $46°$. The angles are equal in measure because a sequence of rotations preserves the degrees of an angle.

c. What is the length of segment DE, and how does it compare to the length of segment $D'E'$? Explain.

The length of segment DE is 4 cm. The length of segment $D'E'$ is also 4 cm. The segments are equal in length because a sequence of rotations preserves the length of segments.

2. Refer to the figure given below.

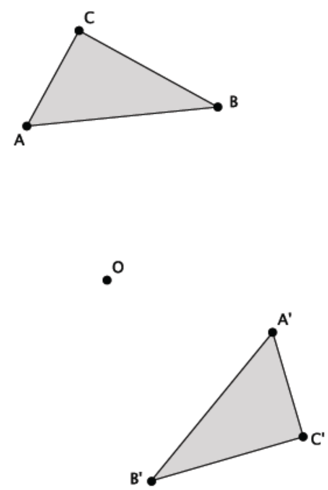


a. Let $Rotation_1$ be a counterclockwise rotation of $90°$ around the center O. Let $Rotation_2$ be a clockwise rotation of $(-45)°$ around the center Q. Determine the approximate location of $Rotation_1(\triangle ABC)$ followed by $Rotation_2$. Label the image of $\triangle ABC$ as $\triangle A'B'C'$.

b. **Describe the sequence of rigid motions that would map $\triangle ABC$ onto $\triangle A'B'C'$.**

The image of $\triangle ABC$ is shown above. Translate $\triangle ABC$ along vector $\overrightarrow{AA'}$. Rotate $\triangle ABC$ d degrees around center A'. Then, $\triangle ABC$ will map onto $\triangle A'B'C'$.

3. **Refer to the figure given below.**



Let R be a rotation of $(-90)°$ around the center O. Let $Rotation_2$ be a rotation of $(-45)°$ around the same center O. Determine the approximate location of $Rotation_1(\triangle ABC)$ followed by $Rotation_2(\triangle ABC)$. Label the image of $\triangle ABC$ as $\triangle A'B'C'$.

The image of $\triangle ABC$ is shown above.