Lesson 17: Equations Involving Factored Expressions

Student Outcomes

- Students learn that equations of the form $(x-a)(x-b)=0$ have the same solution set as two equations joined by "or": $x-a=0$ or $x-b=0$. Students solve factored or easily factorable equations.

Classwork

Exercise 1 (5 minutes)

Allow students a few minutes to complete only parts (a) through (d) of Exercise 1, either individually or in pairs.

MP.7 & MP.8

Exercise 1

1. **Solve each equation for x.**

   a. $x-10=0$

   $\{10\}$

   b. $\frac{x}{2}+20=0$

   $\{-40\}$

   c. **Demanding Dwight insists that you give him two solutions to the following equation:**

   $$(x-10)\left(\frac{x}{2}+20\right)=0$$

   Can you provide him with two solutions?

   $\{10,-40\}$

   d. **Demanding Dwight now wants FIVE solutions to the following equation:**

   $$(x-10)(2x+6)(x^2-36)(x^2+10)\left(\frac{x}{2}+20\right)=0$$

   Can you provide him with five solutions?

   $\{-40,-6,-3,6,10\}$

   Do you think there might be a sixth solution?

   There are exactly 5 solutions.

MP.7 & MP.8

Discussion (5 minutes)

- If I told you that the product of two numbers is 20, could you tell me anything about the two numbers?
- Would the numbers have to be 4 and 5?
- Would both numbers have to be smaller than 20?
- Would they both have to be positive?
- Is there much at all you could say about the two numbers?
  - *Not really. They have to have the same sign is about all we can say.*
- If I told you that the product of two numbers is zero, could you tell me anything about the two numbers?
  - *At least one of the numbers must be zero.*
- How could we phrase this mathematically?
  - *If $ab = 0$, then either $a = 0$ or $b = 0$ or $a = b = 0$.*
- This is known as the **zero-product property**.
- What if the product of three numbers is zero? What if the product of seven numbers is zero?
  - *If any product of numbers is zero, at least one of the terms in that product is zero.*

Exercise 1 (continued) (2 minutes)

Give students a few minutes to complete parts (e) and (f), and elicit student responses.

Consider the equation $(x-4)(x+3) = 0$.

e. Rewrite the equation as a compound statement.

$x - 4 = 0$ ***or*** $x + 3 = 0$

f. Find the two solutions to the equation.

$\{-3, 4\}$

Scaffolding:

Give early finishers this challenge: Write a factored equation that has the solution: $\{-5, -4\}$.

Examples 1–2 (5 minutes)

Work the two examples as a class.

Example 1

Solve $2x^2 - 10x = 0$, for x.

$\{0, 5\}$

Example 2

Solve $x(x-3) + 5(x-3) = 0$, for x.

$\{-5, 3\}$

Scaffolding:

Remind students of the practice of applying the distribution property "backward" that they saw in the Lesson 6 Problem Set. This practice is called *factoring*.

MP.6

Lead a discussion about the application of the distributive property, in the form of factoring polynomial expressions, when solving the equations in these two examples.

Students may want to divide both sides by x. Remind them that x is an unknown quantity that could be positive, negative, or zero. These cases need to be handled separately to get the correct answer. Here we will take a more familiar approach in the solution process: factoring.

Continue to emphasize the idea of rewriting the factored equation as a compound statement. Do not let students skip this step!

Exercises 2–7 (7 minutes)

Give students time to work on the problems individually. As students finish, have them work the problems on the board. Answers are below.

Exercises 2–7

2. $(x+1)(x+2)=0$

   $\{-2,-1\}$

3. $(3x-2)(x+12)=0$

   $\left\{-12,\frac{2}{3}\right\}$

4. $(x-3)(x-3)=0$

   $\{3\}$

5. $(x+4)(x-6)(x-10)=0$

   $\{-4,6,10\}$

6. $x^2-6x=0$

   $\{0,6\}$

7. $x(x-5)+4(x-5)=0$

   $\{-4,5\}$

Example 3 (3 minutes)

Example 3

Consider the equation $(x-2)(2x-3)=(x-2)(x+5)$. Lulu chooses to multiply through by $\frac{1}{x-2}$ and gets the answer $x=8$. But Poindexter points out that $x=2$ is also an answer, which Lulu missed.

a. **What's the problem with Lulu's approach?**

   You cannot multiply by $\frac{1}{x-2}$ because $x-2$ could equal 0, which means that you would be dividing by 0.

b. **Use factoring to solve the original equation for x.**

$$(x-2)(2x-3)-(x-2)(x+5)=0$$
$$(x-2)\big(2x-3-(x+5)\big)=0$$
$$(x-2)(x-8)=0$$
$$x-2=0 \text{ or } x-8=0$$
$$x=2 \text{ or } x=8$$

Work through the responses as a class.

- Emphasize the idea that multiplying by $\frac{1}{x-2}$ is a problem when $x-2$ equals 0.

Exercises 8–11 (10 minutes)

Give students time to work on Exercises 8–11 in pairs. Then, elicit student responses. Remind students of the danger of multiplying both sides by a variable expression.

> *Scaffolding:*
>
> The problems seen in Exercise 9 are often called *the difference of two squares*. Give early finishers this challenge:
>
> $x^4 - 81 = (x^2)^2 - 9^2 = ?$

Exercises 8–11

8. Use factoring to solve the equation for x: $(x-2)(2x-3) = (x-2)(x+1)$.

$\{2, 4\}$

9. Solve each of the following for x:

a. $x + 2 = 5$

$\{3\}$

b. $x^2 + 2x = 5x$

$\{0, 3\}$

c. $x(5x - 20) + 2(5x - 20) = 5(5x - 20)$

$\{3, 4\}$

10.

a. **Verify:** $(a-5)(a+5) = a^2 - 25$.

$a^2 + 5a - 5a - 25 = a^2 - 25$
$a^2 - 25 = a^2 - 25$

b. **Verify:** $(x-88)(x+88) = x^2 - 88^2$.

$x^2 + 88x - 88x - 88^2 = x^2 - 88^2$
$x^2 - 88^2 = x^2 - 88^2$

c. **Verify:** $A^2 - B^2 = (A-B)(A+B)$.

$A^2 - B^2 = A^2 + AB - AB - B^2$
$A^2 - B^2 = A^2 - B^2$

d. **Solve for x:** $x^2 - 9 = 5(x-3)$.

$\{2, 3\}$

e. **Solve for w:** $(w+2)(w-5) = w^2 - 4$.

$\{-2\}$

11. A string 60 inches long is to be laid out on a tabletop to make a rectangle of perimeter 60 inches. Write the width of the rectangle as $15 + x$ inches. What is an expression for its length? What is an expression for its area? What value for x gives an area of the largest possible value? Describe the shape of the rectangle for this special value of x.

Length: $15 - x$ *Area:* $(15 - x)(15 + x)$

The largest area is when $x = 0$. In this case, the rectangle is a square with length and width both equal to 15.

Discuss the results of Exercise 10.

Work through Exercise 11 as a class, explaining why $x = 0$ gives the largest area.

- Since $(15 + x)(15 - x) = 225 - x^2$ as x gets larger, $225 - x^2$ gets smaller until $x = 15$ at which point the area is zero. So, the domain of x for this problem is $0 \le x \le 15$.
- How can we change the domain if we don't want to allow zero area?
  - *You can leave the 15 end of the interval open if you don't want to allow zero area.*

Closing (3 minutes)

Elicit student responses. Students should make notes of responses in the Lesson Summary rectangle.

- If the product of 4 numbers is zero, what do we know about the numbers? At least one of them must equal 0.
- What is the danger of dividing both sides of an equation by a variable factor? What should be done instead?

> **Lesson Summary**
>
> **The *zero-product property* says that if $ab = 0$, then either $a = 0$ or $b = 0$ or $a = b = 0$.**

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 17: Equations Involving Factored Expressions

Exit Ticket

1. Find the solution set to the equation $3x^2 + 27x = 0$.

2. Determine if each statement is true or false. If the statement is false, explain why, or show work proving that it is false.

   a. If $a = 5$, then $ac = 5c$.

   b. If $ac = 5c$, then $a = 5$.

Exit Ticket Solutions

1. Find the solution set to the equation $3x^2 + 27x = 0$.

   $3x(x+9) = 0$

   $3x = 0$ *or* $x + 9 = 0$

   $x = 0$ *or* $x = -9$

   Solution set: $\{-9, 0\}$

2. Determine if each statement is true or false. If the statement is false, explain why, or show work proving that it is false.

   a. If $a = 5$, then $ac = 5c$.

      True.

   b. If $ac = 5c$, then $a = 5$.

      False. a could equal 5, or c could equal 0.

      $ac = 5c$

      $ac - 5c = 0$

      $c(a-5) = 0$

      $c = 0$ *or* $a - 5 = 0$

      $c = 0$ *or* $a = 5$

Problem Set Solutions

1. Find the solution set of each equation:

   a. $(x-1)(x-2)(x-3) = 0$

      $\{1, 2, 3\}$

   b. $(x-16.5)(x-109) = 0$

      $\{16.5, 109\}$

   c. $x(x+7) + 5(x+7) = 0$

      $\{-7, -5\}$

   d. $x^2 + 8x + 15 = 0$

      $\{-5, -3\}$

   e. $(x-3)(x+3) = 8x$

      $\{-1, 9\}$

2. **Solve $x^2 - 11x = 0$, for x.**

   $\{0, 11\}$

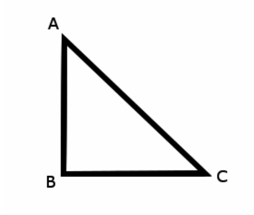
3. **Solve $(p+3)(p-5) = 2(p+3)$, for p. What solution do you lose if you simply divide by $p+3$ to get $p-5=2$?**

   $p = -3$ or $p = 7$. The lost solution is $p = -3$. We assumed $p+3$ was not zero when we divided by $p+3$; therefore, our solution was only complete for p values not equal to -3.

4. **The square of a number plus 3 times the number is equal to 4. What is the number?**

   Solve $x^2 + 3x = 4$, for x, to obtain $x = -4$ or $x = 1$.

5. **In the right triangle shown below, the length of side AB is x, the length of side BC is $x+2$, and the length of the hypotenuse AC is $x+4$. Use this information to find the length of each side. (Use the Pythagorean theorem to get an equation, and solve for x.)**



   Use the Pythagorean theorem to get the equation $x^2 + (x+2)^2 = (x+4)^2$. This is equivalent to $x^2 - 4x - 12 = 0$, and the solutions are -2 and 6. Choose 6 since x represents a length, and the lengths are

   AB: 6

   BC: 8

   AC: 10

6. **Using what you learned in this lesson, create an equation that has 53 and 22 as its only solutions.**

   $(x-22)(x-53) = 0$