Lesson 10: Converting Repeating Decimals to Fractions

Classwork

Example 1

There is a fraction with an infinite decimal expansion of $0.\overline{81}$. Find the fraction.

Exercises 1–2

1. There is a fraction with an infinite decimal expansion of $0.\overline{123}$. Let $x = 0.\overline{123}$.
   a. Explain why looking at $1000x$ helps us find the fractional representation of x.

b. What is x as a fraction?

c. Is your answer reasonable? Check your answer using a calculator.

2. There is a fraction with a decimal expansion of $0.\overline{4}$. Find the fraction, and check your answer using a calculator.

Example 2

Could it be that $2.13\overline{8}$ is also a fraction?

Exercises 3–4

3. Find the fraction equal to $1.6\overline{23}$. Check your answer using a calculator.

4. Find the fraction equal to $2.9\overline{60}$. Check your answer using a calculator.

Lesson Summary

Every decimal with a repeating pattern is a rational number, and we have the means to determine the fraction that has a given repeating decimal expansion.

Example: Find the fraction that is equal to the number $0.\overline{567}$.

Let x represent the infinite decimal $0.\overline{567}$.

$x = 0.\overline{567}$	
$10^3x = 10^3(0.\overline{567})$	Multiply by 10^3 because there are 3 digits that repeat.
$1000x = 567.\overline{567}$	Simplify
$1000x = 567 + 0.\overline{567}$	By addition
$1000x = 567 + x$	By substitution; $x = 0.\overline{567}$
$1000x - x = 567 + x - x$	Subtraction property of equality
$999x = 567$	Simplify
$\frac{999}{999}x = \frac{567}{999}$	Division property of equality
$x = \frac{567}{999} = \frac{63}{111}$	Simplify

This process may need to be used more than once when the repeating digits, as in numbers such as $1.2\overline{6}$, do not begin immediately after the decimal.

Irrational numbers are numbers that are not rational. They have infinite decimal expansions that do not repeat and they cannot be expressed as $\frac{p}{q}$ for integers p and q with $q \neq 0$.

Problem Set

1.
   a. Let $x = 0.\overline{631}$. Explain why multiplying both sides of this equation by 10^3 will help us determine the fractional representation of x.
   b. What fraction is x?
   c. Is your answer reasonable? Check your answer using a calculator.

2. Find the fraction equal to $3.40\overline{8}$. Check your answer using a calculator.

3. Find the fraction equal to $0.\overline{5923}$. Check your answer using a calculator.

4. Find the fraction equal to $2.3\overline{82}$. Check your answer using a calculator.

5. Find the fraction equal to $0.\overline{714285}$. Check your answer using a calculator.

6. Explain why an infinite decimal that is not a repeating decimal cannot be rational.

7. In a previous lesson, we were convinced that it is acceptable to write $0.\overline{9} = 1$. Use what you learned today to show that it is true.

8. Examine the following repeating infinite decimals and their fraction equivalents. What do you notice? Why do you think what you observed is true?

$0.\overline{81} = \frac{81}{99}$ $\quad$ $0.\overline{4} = \frac{4}{9}$ $\quad$ $0.\overline{123} = \frac{123}{999}$ $\quad$ $0.\overline{60} = \frac{60}{99}$

$0.\overline{4311} = \frac{4311}{9999}$ $\quad$ $0.\overline{01} = \frac{1}{99}$ $\quad$ $0.\overline{3} = \frac{1}{3} = \frac{3}{9}$ $\quad$ $0.\overline{9} = 1.0$