Lesson 8: The Long Division Algorithm

Classwork

Example 1

Show that the decimal expansion of $\frac{26}{4}$ is 6.5.

Exploratory Challenge/Exercises 1–5

1.

a. Use long division to determine the decimal expansion of $\frac{142}{2}$.

b. Fill in the blanks to show another way to determine the decimal expansion of $\frac{142}{2}$.

$$142 = \_\_\_\_ \times 2 + \_\_\_\_$$

$$\frac{142}{2} = \frac{\_\_\_\_ \times 2 + \_\_\_\_}{2}$$

$$\frac{142}{2} = \frac{\_\_\_\_ \times 2}{2} + \frac{\_\_\_\_}{2}$$

$$\frac{142}{2} = \_\_\_\_ + \frac{\_\_\_\_}{2}$$

$$\frac{142}{2} = \_\_\_\_\_\_$$

c. Does the number $\frac{142}{2}$ have a finite or an infinite decimal expansion?

2.

a. Use long division to determine the decimal expansion of $\frac{142}{4}$.

b. Fill in the blanks to show another way to determine the decimal expansion of $\frac{142}{4}$.

$$142 = \_\_\_\_ \times 4 + \_\_\_\_$$

$$\frac{142}{4} = \frac{\_\_\_\_ \times 4 + \_\_\_\_}{4}$$

$$\frac{142}{4} = \frac{\_\_\_\_ \times 4}{4} + \frac{\_\_\_\_}{4}$$

$$\frac{142}{4} = \_\_\_\_ + \frac{\_\_\_\_}{4}$$

$$\frac{142}{4} = \_\_\_\_\_\_\_\_$$

c. Does the number $\frac{142}{4}$ have a finite or an infinite decimal expansion?

3.

a. Use long division to determine the decimal expansion of $\frac{142}{6}$.

b. Fill in the blanks to show another way to determine the decimal expansion of $\frac{142}{6}$.

$$142 = \_\_\_\_ \times 6 + \_\_\_\_$$

$$\frac{142}{6} = \frac{\_\_\_\_ \times 6 + \_\_\_\_}{6}$$

$$\frac{142}{6} = \frac{\_\_\_\_ \times 6}{6} + \frac{\_\_\_\_}{6}$$

$$\frac{142}{6} = \_\_\_\_ + \frac{\_\_\_\_}{6}$$

$$\frac{142}{6} = \_\_\_\_\_\_\_\_$$

c. Does the number $\frac{142}{6}$ have a finite or an infinite decimal expansion?

4.

a. Use long division to determine the decimal expansion of $\frac{142}{11}$.

b. Fill in the blanks to show another way to determine the decimal expansion of $\frac{142}{11}$.

$$142 = \_\_\_\_ \times 11 + \_\_\_\_$$

$$\frac{142}{11} = \frac{\_\_\_\_ \times 11 + \_\_\_\_}{11}$$

$$\frac{142}{11} = \frac{\_\_\_\_ \times 11}{11} + \frac{\_\_\_\_}{11}$$

$$\frac{142}{11} = \_\_\_\_ + \frac{\_\_\_\_}{11}$$

$$\frac{142}{11} = \_\_\_\_\_\_\_\_\_\_\_\_$$

c. Does the number $\frac{142}{11}$ have a finite or an infinite decimal expansion?

5. In general, which fractions produce infinite decimal expansions?

Exercises 6–10

6. Does the number $\frac{65}{13}$ have a finite or an infinite decimal expansion? Does its decimal expansion have a repeating pattern?

7. Does the number $\frac{17}{11}$ have a finite or an infinite decimal expansion? Does its decimal expansion have a repeating pattern?

8. Is the number $0.212112111211112111112\ldots$ rational? Explain. (Assume the pattern you see in the decimal expansion continues.)

9. Does the number $\frac{860}{999}$ have a finite or an infinite decimal expansion? Does its decimal expansion have a repeating pattern?

10. Is the number $0.1234567891011121314151617181920212223\ldots$ rational? Explain. (Assume the pattern you see in the decimal expansion continues.)

Lesson Summary

A rational number is a number that can be written in the form $\frac{a}{b}$ for a pair of integers a and b with b not zero.

The long division algorithm shows that every rational number has a decimal expansion that falls into a repeating pattern. For example, the rational number 32 has a decimal expansion of $32.\overline{0}$, the rational number $\frac{1}{3}$ has a decimal expansion of $0.\overline{3}$, and the rational number $\frac{4}{11}$ has a decimal expansion of $0.\overline{45}$.

Problem Set

1. Write the decimal expansion of $\frac{7000}{9}$ as an infinitely long repeating decimal.

2. Write the decimal expansion of $\frac{6555555}{3}$ as an infinitely long repeating decimal.

3. Write the decimal expansion of $\frac{350000}{11}$ as an infinitely long repeating decimal.

4. Write the decimal expansion of $\frac{1200000}{37}$ as an infinitely long repeating decimal.

5. Someone notices that the long division of $2{,}222{,}222$ by 6 has a quotient of $370{,}370$ and a remainder of 2 and wonders why there is a repeating block of digits in the quotient, namely 370. Explain to the person why this happens.

6. Is the answer to the division problem number $10 \div 3.2$ a rational number? Explain.

7. Is $\frac{3\pi}{77\pi}$ a rational number? Explain.

8. The decimal expansion of a real number x has every digit 0 except the first digit, the tenth digit, the hundredth digit, the thousandth digit, and so on, are each 1. Is x a rational number? Explain.