

Lesson 2: Interpreting Rate of Change and Initial Value

Classwork

Linear functions are defined by the equation of a line. The graphs and the equations of the lines are important for understanding the relationship between the two variables represented in the following example as x and y .

Example 1: Rate of Change and Initial Value

The equation of a line can be interpreted as defining a linear function. The graphs and the equations of lines are important in understanding the relationship between two types of quantities (represented in the following examples by x and y).

In a previous lesson, you encountered an MP3 download site that offers downloads of individual songs with the following price structure: a \$3 fixed fee for a monthly subscription *plus* a fee of \$0.25 per song. The linear function that models the relationship between the number of songs downloaded and the total monthly cost of downloading songs can be written as

$$y = 0.25x + 3,$$

where x represents the number of songs downloaded and y represents the total monthly cost (in dollars) for MP3 downloads.

- a. In your own words, explain the meaning of 0.25 within the context of the problem.

- b. In your own words, explain the meaning of 3 within the context of the problem.

The values represented in the function can be interpreted in the following way:

$$y = \underbrace{0.25x}_{\text{rate of change}} + \underbrace{3}_{\text{initial value}}$$

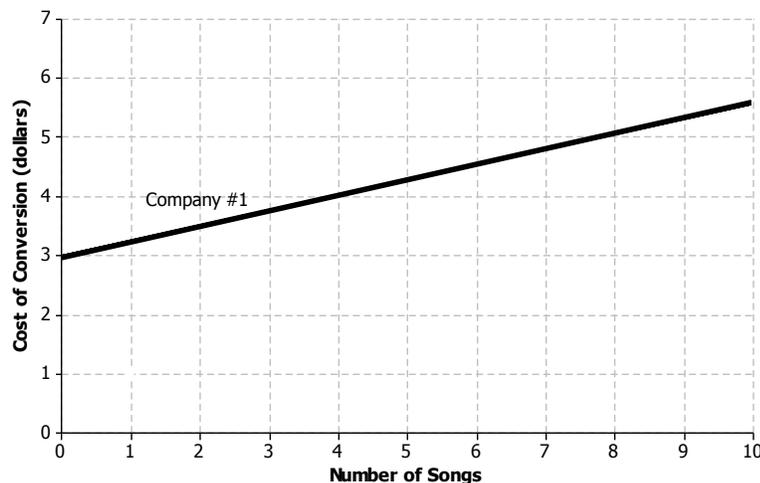
The coefficient of x is referred to as the *rate of change*. It can be interpreted as the change in the values of y for every one-unit increase in the values of x . When the rate of change is positive, the linear function is *increasing*. In other words, *increasing* indicates that as the x -value increases, so does the y -value. When the rate of change is negative, the linear function is *decreasing*. *Decreasing* indicates that as the x -value increases, the y -value decreases.

The constant value is referred to as the *initial value* or y -intercept and can be interpreted as the value of y when $x = 0$.

Exercises 1–6: Is It a Better Deal?

Another site offers MP3 downloads with a different price structure: a \$2 fixed fee for a monthly subscription *plus* a fee of \$0.40 per song.

- Write a linear function to model the relationship between the number of songs downloaded and the total monthly cost. As before, let x represent the number of songs downloaded and y represent the total monthly cost (in dollars) of downloading songs.
- Determine the cost of downloading 0 songs and 10 songs from this site.
- The graph below already shows the linear model for the first subscription site (Company 1): $y = 0.25x + 3$. Graph the equation of the line for the second subscription site (Company 2) by marking the two points from your work in Exercise 2 (for 0 songs and 10 songs) and drawing a line through those two points.



4. Which line has a steeper slope? Which company's model has the more expensive cost per song?

5. Which function has the greater initial value?

6. Which subscription site would you choose if you only wanted to download 5 songs per month? Which company would you choose if you wanted to download 10 songs? Explain your reasoning.

Exercises 7–9: Aging Autos

7. When someone purchases a new car and begins to drive it, the mileage (meaning the number of miles the car has traveled) immediately increases. Let x represent the number of years since the car was purchased and y represent the total miles traveled. The linear function that models the relationship between the number of years since purchase and the total miles traveled is $y = 15000x$.
 - a. Identify and interpret the rate of change.

 - b. Identify and interpret the initial value.

- c. Is the mileage increasing or decreasing each year according to the model? Explain your reasoning.
8. When someone purchases a new car and begins to drive it, generally speaking, the resale value of the car (in dollars) goes down each year. Let x represent the number of years since purchase and y represent the resale value of the car (in dollars). The linear function that models the resale value based on the number of years since purchase is $y = 20000 - 1200x$.
- a. Identify and interpret the rate of change.
- b. Identify and interpret the initial value.
- c. Is the resale value increasing or decreasing each year according to the model? Explain.
9. Suppose you are given the linear function $y = 2.5x + 10$.
- a. Write a story that can be modeled by the given linear function.
- b. What is the rate of change? Explain its meaning with respect to your story.
- c. What is the initial value? Explain its meaning with respect to your story.

Lesson Summary

When a linear function is given by the equation of a line of the form $y = mx + b$, the rate of change is m , and the initial value is b . Both are easy to identify.

The rate of change of a linear function is the slope of the line it represents. It is the change in the values of y per a one-unit increase in the values of x .

- A positive rate of change indicates that a linear function is increasing.
- A negative rate of change indicates that a linear function is decreasing.
- Given two lines each with positive slope, the function represented by the steeper line has a greater rate of change.

The initial value of a linear function is the value of the y -variable when the x -value is zero.

Problem Set

1. A rental car company offers the following two pricing methods for its customers to choose from for a one-month rental:

Method 1: Pay \$400 for the month, or

Method 2: Pay \$0.30 per mile plus a standard maintenance fee of \$35.

- a. Construct a linear function that models the relationship between the miles driven and the total rental cost for Method 2. Let x represent the number of miles driven and y represent the rental cost (in dollars).
 - b. If you plan to drive 1,100 miles for the month, which method would you choose? Explain your reasoning.
2. Recall from a previous lesson that Kelly wants to add new music to her MP3 player. She was interested in a monthly subscription site that offered its MP3 downloading service for a monthly subscription fee *plus* a fee per song. The linear function that modeled the total monthly cost in dollars (y) based on the number of songs downloaded (x) is $y = 5.25 + 0.30x$.

The site has suddenly changed its monthly price structure. The linear function that models the new total monthly cost in dollars (y) based on the number of songs downloaded (x) is $y = 0.35x + 4.50$.

- a. Explain the meaning of the value 4.50 in the new equation. Is this a better situation for Kelly than before?
- b. Explain the meaning of the value 0.35 in the new equation. Is this a better situation for Kelly than before?
- c. If you were to graph the two equations (old versus new), which line would have the steeper slope? What does this mean in the context of the problem?
- d. Which subscription plan provides the better value if Kelly downloads fewer than 15 songs per month?