



Lesson 1: Modeling Linear Relationships

Student Outcomes

- Students determine a linear function given a verbal description of a linear relationship between two quantities.
- Students interpret linear functions based on the context of a problem.
- Students sketch the graph of a linear function by constructing a table of values, plotting points, and connecting points by a line.

Lesson Notes

In this first lesson, students construct linear functions based on verbal descriptions of bivariate data. They graph the linear functions by creating a table of values, plotting points, and drawing the line. Throughout this lesson, provide students with the opportunity to explain the functions in terms of the equation of the line and the relationship between the two variables. Emphasize the context with students as they explain the rates of change and the initial values.

Classwork

Example 1 (2–3 minutes): Logging On

Read through the example as a class. Convey to students that the information presented in the example can be organized into ordered pairs or points. Minutes can be represented by x and cost by y .

Example 1: Logging On

Lenore has just purchased a tablet computer, and she is considering purchasing an Internet access plan so that she can connect to the Internet wirelessly from virtually anywhere in the world. One company offers an Internet access plan so that when a person connects to the company's wireless network, the person is charged a fixed access fee for connecting *plus* an amount for the number of minutes connected based upon a constant usage rate in dollars per minute.

Lenore is considering this company's plan, but the company's advertisement does not state how much the fixed access fee for connecting is, nor does it state the usage rate. However, the company's website says that a 10-minute session costs \$0.40, a 20-minute session costs \$0.70, and a 30-minute session costs \$1.00. Lenore decides to use these pieces of information to determine both the fixed access fee for connecting and the usage rate.

Exercises 1–6 (10 minutes)

This exercise set introduces students to constant rate of change and initial value and how those values are used to construct a function to model a situation. Pose each exercise to the class, one at a time, using the following questions to encourage discussion.

After Exercise 1, discuss as a class the need to graph this real-world problem only in the first quadrant. Begin by asking students the following:

- If we used the entire coordinate plane to graph this line, what would the negative x values represent?
 - *The x -axis represents minutes. So, time would be negative, which does not make sense in the context of the problem.*

For Exercise 2, use the table to demonstrate constant rate of change to students.

- How could we use the table to determine the constant rate of change?

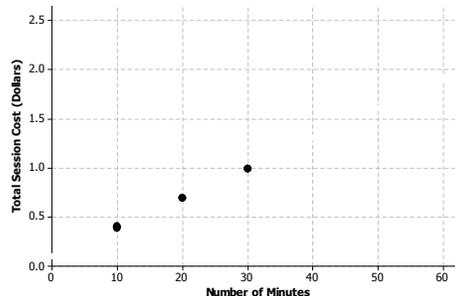
| Number of Minutes | Total Session Cost in Dollars |
|-------------------|-------------------------------|
| 0 | |
| 10 | |
| 20 | 0.40 |
| 30 | 0.70 |
| 40 | 1.00 |
| 50 | |
| 60 | |

Exercises 1–6

- Lenore makes a table of this information and a graph where number of minutes is represented by the horizontal axis and total session cost is represented by the vertical axis. Plot the three given points on the graph. These three points appear to lie on a line. What information about the access plan suggests that the correct model is indeed a linear relationship?

The amount charged for the minutes connected is based upon a constant usage rate in dollars per minute.

| Number of Minutes | Total Session Cost in Dollars |
|-------------------|-------------------------------|
| 0 | |
| 10 | |
| 20 | 0.40 |
| 30 | 0.70 |
| 40 | 1.00 |
| 50 | |
| 60 | |



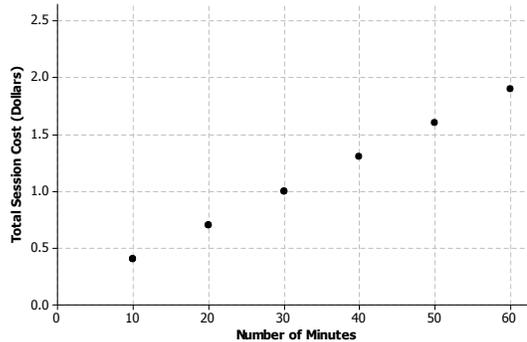
- The rate of change describes how the total cost changes with respect to time.
 - When the number of minutes increases by 10 (e.g., from 10 minutes to 20 minutes or from 20 minutes to 30 minutes), how much does the charge increase?
When the number of minutes increases by 10 (e.g., from 10 minutes to 20 minutes or from 20 minutes to 30 minutes), the cost increases by \$0.30 (30 cents).
 - Another way to say this would be the usage charge per 10 minutes of use. Use that information to determine the increase in cost based on only 1 minute of additional usage. In other words, find the usage charge per minute of use.

If \$0.30 is the usage charge per 10 minutes of use, then \$0.03 is the usage charge per 1 minute of use (i.e., the usage rate). Since the usage rate is constant, students should use what they have learned in Module 4.

3. The company’s pricing plan states that the usage rate is constant for any number of minutes connected to the Internet. In other words, the increase in cost for 10 more minutes of use (the value that you calculated in Exercise 2) is the same whether you increase from 20 to 30 minutes, 30 to 40 minutes, etc. Using this information, determine the total cost for 40 minutes, 50 minutes, and 60 minutes of use. Record those values in the table, and plot the corresponding points on the graph in Exercise 1.

Consider the following table and graphs.

| Number of Minutes | Total Session Cost (in dollars) |
|-------------------|---------------------------------|
| 0 | |
| 10 | 0.40 |
| 20 | 0.70 |
| 30 | 1.00 |
| 40 | 1.30 |
| 50 | 1.60 |
| 60 | 1.90 |



4. Using the table and the graph in Exercise 1, compute the hypothetical cost for 0 minutes of use. What does that value represent in the context of the values that Lenore is trying to figure out?

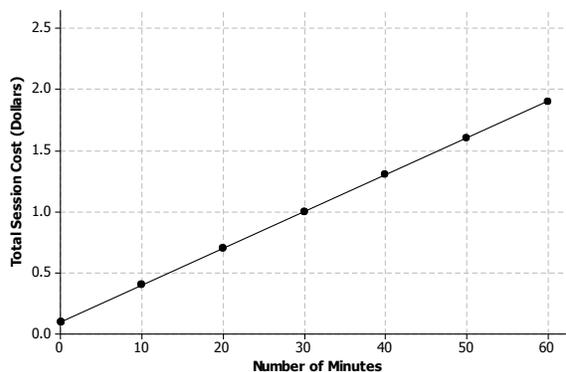
Since there is a \$0.30 decrease in cost for each decrease of 10 minutes of use, one could subtract \$0.30 from the cost value for 10 minutes and arrive at the hypothetical cost value for 0 minutes. That cost would be \$0.10. Students may notice that such a value follows the regular pattern in the table and would represent the fixed access fee for connecting. (This value could also be found from the graph after completing Exercise 6.)

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Convey to students that this is known as the initial value.

- Why is this a hypothetical cost?
 - Because it is impossible to connect for 0 minutes; the connection is always for some interval of time.

5. On the graph in Exercise 1, draw a line through the points representing 0 to 60 minutes of use under this company’s plan. The slope of this line is equal to the constant rate of change, which in this case is the usage rate.



6. Using x for the number of minutes and y for the total cost in dollars, write a function to model the linear relationship between minutes of use and total cost.

$$y = 0.03x + 0.10$$



Example 2 (2–3 minutes): Another Rate Plan

Provide students time to read the example. As a whole group, summarize this alternative rate plan.

Example 2: Another Rate Plan

A second wireless access company has a similar method for computing its costs. Unlike the first company that Lenore was considering, this second company explicitly states its access fee is \$0.15, and its usage rate is \$0.04 per minute.

$$\text{Total Session Cost} = \$0.15 + \$0.04 (\text{number of minutes})$$

- How is this plan presented differently?
 - *In this case, we are given the access fee and usage rate with an equation. In the first example, just data points were given.*
- Based on the work with the first set of problems, how do you think the two plans are different?
 - *The values for the access fee and usage charge per minute are different, or the initial value and the rate of change are different.*

Exercises 7–9 (7 minutes)

Allow students to work independently on these exercises. After most students have completed the problems, discuss them as a whole group.

Exercises 7–16

7. Let x represent the number of minutes used and y represent the total session cost in dollars. Construct a linear function that models the total session cost based on the number of minutes used.

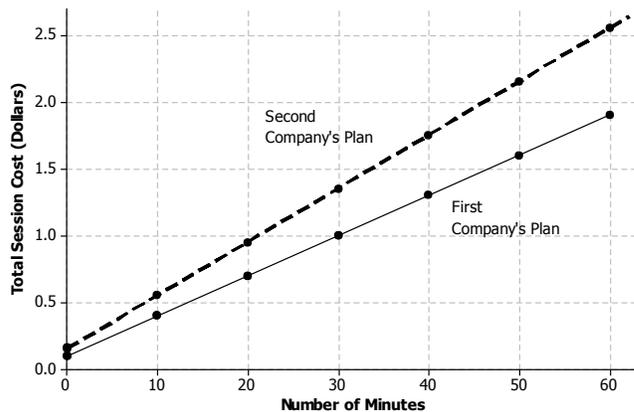
$$y = 0.04x + 0.15$$

8. Using the linear function constructed in Exercise 7, determine the total session cost for sessions of 0, 10, 20, 30, 40, 50, and 60 minutes, and fill in these values in the table below.

| Number of Minutes | Total Session Cost (in dollars) |
|-------------------|---------------------------------|
| 0 | 0.15 |
| 10 | 0.55 |
| 20 | 0.95 |
| 30 | 1.35 |
| 40 | 1.75 |
| 50 | 2.15 |
| 60 | 2.55 |

9. Plot these points on the original graph in Exercise 1, and draw a line through these points. In what ways does the line that represents this second company’s access plan differ from the line that represents the first company’s access plan?

The second company’s plan line begins at a greater initial value. The same plan also increases in total cost more quickly over time; in other words, the slope of the line for the second company’s plan is steeper.



Exercises 10–12 (7 minutes)

MP3 download sites are a popular forum for selling music. Different sites offer pricing that depends on whether or not you want to purchase an entire album or individual songs à la carte. One site offers MP3 downloads of individual songs with the following price structure: a \$3 fixed fee for a monthly subscription *plus* a charge of \$0.25 per song.

10. Using x for the number of songs downloaded and y for the total monthly cost in dollars, construct a linear function to model the relationship between the number of songs downloaded and the total monthly cost.

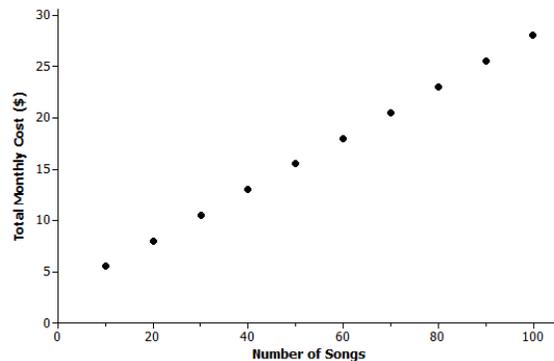
Since \$3 is the initial cost and there is a 25 cent increase per song, the function would be

$y = 3 + 0.25x$ or $y = 0.25x + 3$.

11. Using the linear function you wrote in Exercise 10, construct a table to record the total monthly cost (in dollars) for MP3 downloads of 10 songs, 20 songs, and so on up to 100 songs.

| Number of Songs | Total Monthly Cost (in dollars) |
|-----------------|---------------------------------|
| 10 | 5.50 |
| 20 | 8.00 |
| 30 | 10.50 |
| 40 | 13.00 |
| 50 | 15.50 |
| 60 | 18.00 |
| 70 | 20.50 |
| 80 | 23.00 |
| 90 | 25.50 |
| 100 | 28.00 |

12. Plot the 10 data points in the table on a coordinate plane. Let the x -axis represent the number of songs downloaded and the y -axis represent the total monthly cost (in dollars) for MP3 downloads.



Exercises 13–16 (7–8 minutes)

Read through the problem as a class. The data in this exercise set are presented as two points given in context. Point out the difference by asking students the following:

- How are the data in this problem different from the data in the MP3 problem?
 - *In this problem, the data can be organized as ordered pairs. In the MP3 problem, a rate of change and initial value were given.*

A band will be paid a flat fee for playing a concert. Additionally, the band will receive a fixed amount for every ticket sold. If 40 tickets are sold, the band will be paid \$200. If 70 tickets are sold, the band will be paid \$260.

13. Determine the rate of change.

The points (40, 200) and (70, 260) have been given.

So, the rate of change is 2 because $\frac{260 - 200}{70 - 40} = 2$.

14. Let x represent the number of tickets sold and y represent the amount the band will be paid in dollars. Construct a linear function to represent the relationship between the number of tickets sold and the amount the band will be paid.

Using the rate of change and (40, 200):

$$200 = 2(40) + b$$

$$200 = 80 + b$$

$$120 = b$$

Therefore, the function is $y = 2x + 120$.

15. What flat fee will the band be paid for playing the concert regardless of the number of tickets sold?

The band will be paid a flat fee of \$120 for playing the concert.

16. How much will the band receive for each ticket sold?

The band receives \$2 per ticket.

Closing (5 minutes)

Consider posing the following questions; allow a few student responses for each.

- In Exercise 9 when the two pricing models that Lenore was considering were both displayed on the same graph, was there ever a point at which the second company's model was a better, less expensive choice than the first company's model?
 - *No. The second company always had the more expensive plan; its line was always above the other company's line.*
- When comparing the equations of the two models, which value in the second company's model (the \$0.15 access fee or \$0.04 cost per minute) led you to think that it would increase at a faster rate than the first model?
 - *The \$0.04 cost per minute led me to believe it would increase at a faster rate. The other company's plan only increased at a rate of \$0.03 per minute.*

Lesson Summary

A linear function can be used to model a linear relationship between two types of quantities. The graph of a linear function is a straight line.

A linear function can be constructed using a rate of change and an initial value. It can be interpreted as an equation of a line in which:

- The rate of change is the slope of the line and describes how one quantity changes with respect to another quantity.
- The initial value is the y -intercept.

Exit Ticket (5 minutes)



Exit Ticket Sample Solutions

A rental car company offers a rental package for a midsize car. The cost comprises a fixed \$30 administrative fee for the cleaning and maintenance of the car plus a rental cost of \$35 per day.

- Using x for the number of days and y for the total cost in dollars, construct a function to model the relationship between the number of days and the total cost of renting a midsize car.

$$y = 35x + 30$$

- The same company is advertising a deal on compact car rentals. The linear function $y = 30x + 15$ can be used to model the relationship between the number of days, x , and the total cost in dollars, y , of renting a compact car.

- What is the fixed administrative fee?

The administrative fee is \$15.

- What is the rental cost per day?

It costs \$30 per day to rent the compact car.

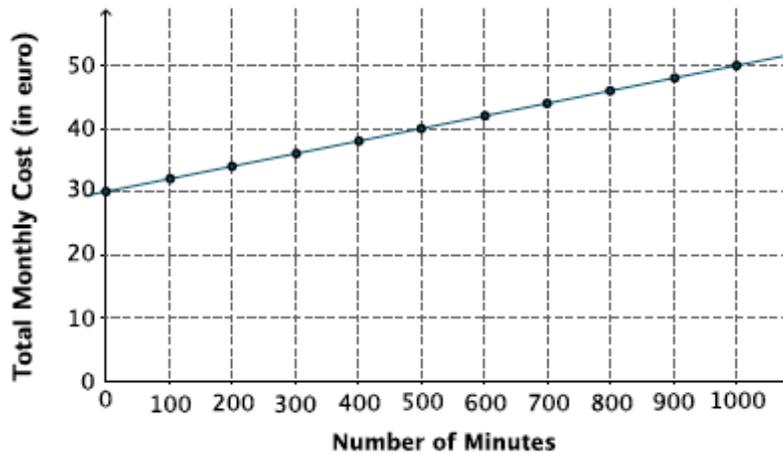
Problem Set Sample Solutions

- Recall that Lenore was investigating two wireless access plans. Her friend in Europe says that he uses a plan in which he pays a monthly fee of 30 euro plus 0.02 euro per minute of use.

- Construct a table of values for his plan's monthly cost based on 100 minutes of use for the month, 200 minutes of use, and so on up to 1,000 minutes of use. (The charge of 0.02 euro per minute of use is equivalent to 2 euro per 100 minutes of use.)

| <i>Number of Minutes</i> | <i>Total Monthly Cost (€)</i> |
|--------------------------|-------------------------------|
| 100 | 32.00 |
| 200 | 34.00 |
| 300 | 36.00 |
| 400 | 38.00 |
| 500 | 40.00 |
| 600 | 42.00 |
| 700 | 44.00 |
| 800 | 46.00 |
| 900 | 48.00 |
| 1,000 | 50.00 |

- b. Plot these 10 points on a carefully labeled graph, and draw the line that contains these points.



- c. Let x represent minutes of use and y represent the total monthly cost in euro. Construct a linear function that determines the monthly cost based on minutes of use.

$$y = 30 + 0.02x$$

- d. Use the function to calculate the cost under this plan for 750 minutes of use. If this point were added to the graph, would it be above the line, below the line, or on the line?

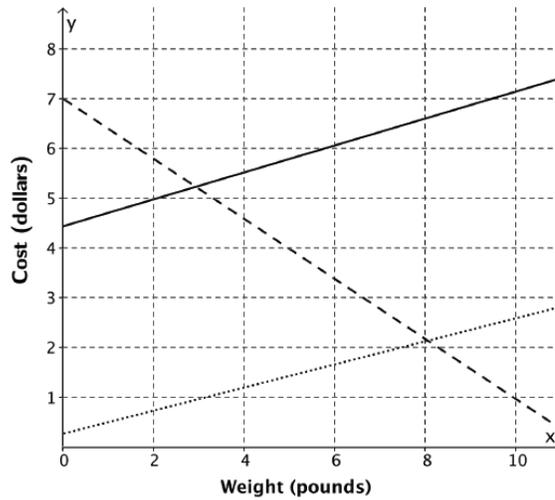
The cost for 750 minutes would be €45. The point (750, 45) would be on the line.

2. A shipping company charges a \$4.45 handling fee in addition to \$0.27 per pound to ship a package.

- a. Using x for the weight in pounds and y for the cost of shipping in dollars, write a linear function that determines the cost of shipping based on weight.

$$y = 4.45 + 0.27x$$

- b. Which line (solid, dotted, or dashed) on the following graph represents the shipping company’s pricing method? Explain.



The solid line would be the correct line. Its initial value is 4.45, and its slope is 0.27. The dashed line shows the cost decreasing as the weight increases, so that is not correct. The dotted line starts at an initial value that is too low.

3. Kelly wants to add new music to her MP3 player. Another subscription site offers its downloading service using the following: Total Monthly Cost = 5.25 + 0.30(number of songs).

- a. Write a sentence (all words, no math symbols) that the company could use on its website to explain how it determines the price for MP3 downloads for the month.

“We charge a \$5.25 subscription fee plus 30 cents per song downloaded.”

- b. Let x represent the number of songs downloaded and y represent the total monthly cost in dollars. Construct a function to model the relationship between the number of songs downloaded and the total monthly cost.

$$y = 5.25 + 0.30x$$

- c. Determine the cost of downloading 10 songs.

$$5.25 + 0.30(10) = 8.25$$

The cost of downloading 10 songs is \$8.25.

4. Li Na is saving money. Her parents gave her an amount to start, and since then she has been putting aside a fixed amount each week. After six weeks, Li Na has a total of \$82 of her own savings in addition to the amount her parents gave her. Fourteen weeks from the start of the process, Li Na has \$118.

- a. Using x for the number of weeks and y for the amount in savings (in dollars), construct a linear function that describes the relationship between the number of weeks and the amount in savings.

The points (6, 82) and (14, 118) have been given.

So, the rate of change is 4.5 because $\frac{118 - 82}{14 - 6} = \frac{36}{8} = 4.5$.

Using the rate of change and (6, 82):

$$82 = 4.5(6) + b$$

$$82 = 27 + b$$

$$55 = b$$

The function is $y = 4.5x + 55$.

- b. How much did Li Na's parents give her to start?

Li Na's parents gave her \$55 to start.

- c. How much does Li Na set aside each week?

Li Na is setting aside \$4.50 every week for savings.

- d. Draw the graph of the linear function below (start by plotting the points for $x = 0$ and $x = 20$).

