

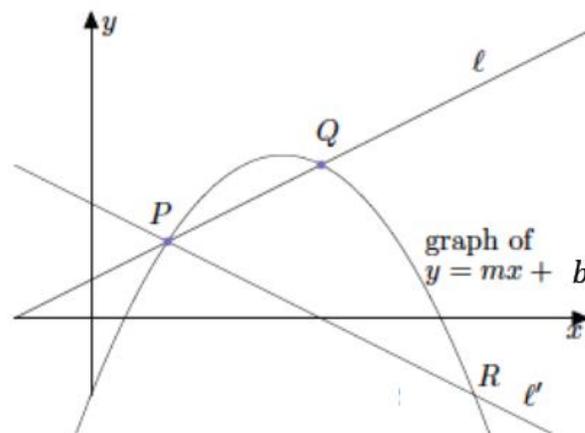
Lesson 19: The Graph of a Linear Equation in Two Variables Is a Line

Classwork

Exercises

THEOREM: The graph of a linear equation $y = mx + b$ is a non-vertical line with slope m and passing through $(0, b)$, where b is a constant.

- Prove the theorem by completing parts (a)–(c). Given two distinct points, P and Q , on the graph of $y = mx + b$, and let l be the line passing through P and Q . You must show the following:
 - Any point on the graph of $y = mx + b$ is on line l , and
 - Any point on the line l is on the graph of $y = mx + b$.
 - Proof of (1): Let R be any point on the graph of $y = mx + b$. Show that R is on l . Begin by assuming it is not. Assume the graph looks like the diagram below where R is on l' .

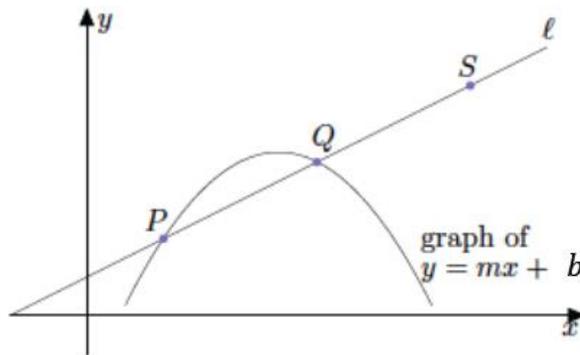


What is the slope of line l ?

What is the slope of line l' ?

What can you conclude about lines l and l' ? Explain.

b. Proof of (2): Let S be any point on line l , as shown.



Show that S is a solution to $y = mx + b$. Hint: Use the point $(0, b)$.

- c. Now that you have shown that any point on the graph of $y = mx + b$ is on line l in part (a), and any point on line l is on the graph of $y = mx + b$ in part (b), what can you conclude about the graphs of linear equations?
2. Use $x = 4$ and $x = -4$ to find two solutions to the equation $x + 2y = 6$. Plot the solutions as points on the coordinate plane, and connect the points to make a line.
- a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation $x + 2y = 6$.
- b. When $x = 1$, what is the value of y ? Does this solution appear to be a point on the line?
- c. When $x = -3$, what is the value of y ? Does this solution appear to be a point on the line?
- d. Is the point $(3, 2)$ on the line?
- e. Is the point $(3, 2)$ a solution to the linear equation $x + 2y = 6$?

3. Use $x = 4$ and $x = 1$ to find two solutions to the equation $3x - y = 9$. Plot the solutions as points on the coordinate plane, and connect the points to make a line.
- Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation $3x - y = 9$.
 - When $x = 4.5$, what is the value of y ? Does this solution appear to be a point on the line?
 - When $x = \frac{1}{2}$, what is the value of y ? Does this solution appear to be a point on the line?
 - Is the point $(2, 4)$ on the line?
 - Is the point $(2, 4)$ a solution to the linear equation $3x - y = 9$?
4. Use $x = 3$ and $x = -3$ to find two solutions to the equation $2x + 3y = 12$. Plot the solutions as points on the coordinate plane, and connect the points to make a line.
- Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation $2x + 3y = 12$.

- b. When $x = 2$, what is the value of y ? Does this solution appear to be a point on the line?
- c. When $x = -2$, what is the value of y ? Does this solution appear to be a point on the line?
- d. Is the point $(8, -3)$ on the line?
- e. Is the point $(8, -3)$ a solution to the linear equation $2x + 3y = 12$?
5. Use $x = 4$ and $x = -4$ to find two solutions to the equation $x - 2y = 8$. Plot the solutions as points on the coordinate plane, and connect the points to make a line.
- a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation $x - 2y = 8$.
- b. When $x = 7$, what is the value of y ? Does this solution appear to be a point on the line?

- c. When $x = -3$, what is the value of y ? Does this solution appear to be a point on the line?
- d. Is the point $(-2, -3)$ on the line?
- e. Is the point $(-2, -3)$ a solution to the linear equation $x - 2y = 8$?
6. Based on your work in Exercises 2–5, what conclusions can you draw about the points on a line and solutions to a linear equation?
7. Based on your work in Exercises 2–5, will a point that is not a solution to a linear equation be a point on the graph of a linear equation? Explain.
8. Based on your work in Exercises 2–5, what conclusions can you draw about the graph of a linear equation?

9. Graph the equation $-3x + 8y = 24$ using intercepts.

10. Graph the equation $x - 6y = 15$ using intercepts.

11. Graph the equation $4x + 3y = 21$ using intercepts.

Lesson Summary

The graph of a linear equation is a line. A linear equation can be graphed using two-points: the x -intercept point and the y -intercept point.

Example:

Graph the equation: $2x + 3y = 9$.

Replace x with zero, and solve for y to determine the y -intercept point.

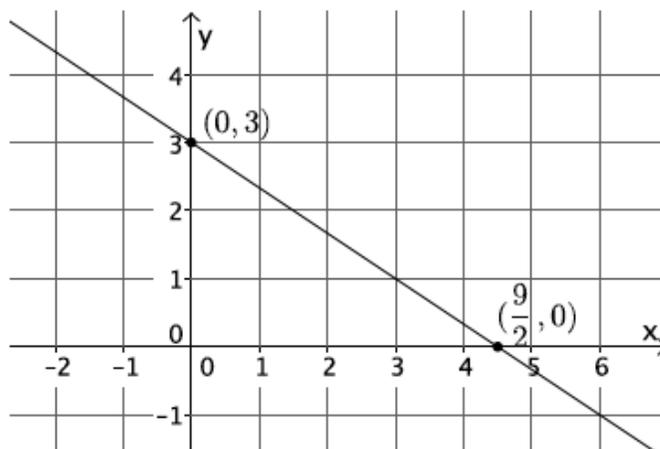
$$\begin{aligned}2(0) + 3y &= 9 \\3y &= 9 \\y &= 3\end{aligned}$$

The y -intercept point is at $(0, 3)$.

Replace y with zero, and solve for x to determine the x -intercept point.

$$\begin{aligned}2x + 3(0) &= 9 \\2x &= 9 \\x &= \frac{9}{2}\end{aligned}$$

The x -intercept point is at $(\frac{9}{2}, 0)$.



Problem Set

Graph each of the equations in the Problem Set on a different pair of x - and y -axes.

1. Graph the equation: $y = -6x + 12$.
2. Graph the equation: $9x + 3y = 18$.
3. Graph the equation: $y = 4x + 2$.
4. Graph the equation: $y = -\frac{5}{7}x + 4$.
5. Graph the equation: $\frac{3}{4}x + y = 8$.
6. Graph the equation: $2x - 4y = 12$.
7. Graph the equation: $y = 3$. What is the slope of the graph of this line?
8. Graph the equation: $x = -4$. What is the slope of the graph of this line?
9. Is the graph of $4x + 5y = \frac{3}{7}$ a line? Explain.
10. Is the graph of $6x^2 - 2y = 7$ a line? Explain.