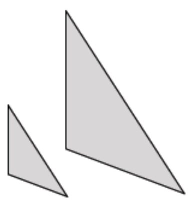
Lesson 1: What Lies Behind "Same Shape"?

Classwork

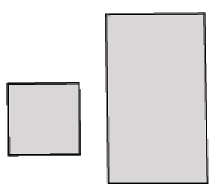
Exploratory Challenge

Two geometric figures are said to be similar if they have the same shape but not necessarily the same size. Using that informal definition, are the following pairs of figures similar to one another? Explain.

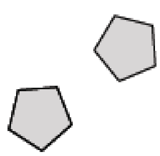
Pair A:



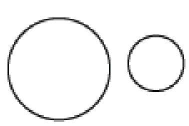
Pair B:



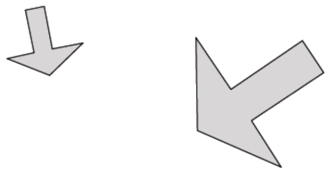
Pair C:



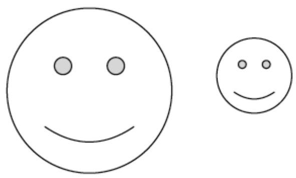
Pair D:



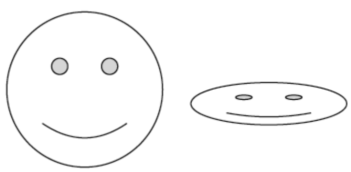
Pair E:



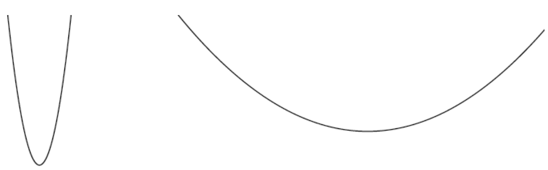
Pair F:



Pair G:



Pair H:



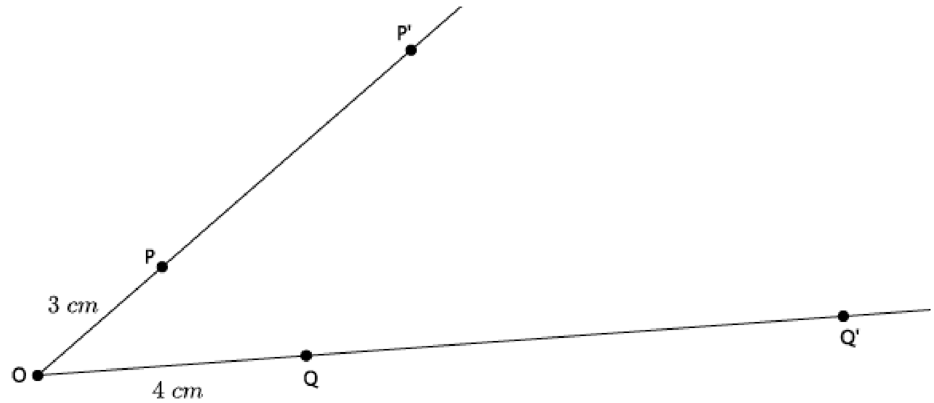
Exercises

1. Given $|OP| = 5$ in.

   a. If segment OP is dilated by a scale factor $r = 4$, what is the length of segment OP'?

   b. If segment OP is dilated by a scale factor $= \frac{1}{2}$, what is the length of segment OP'?

Use the diagram below to answer Exercises 2–6. Let there be a dilation from center O. Then, $Dilation(P) = P'$ and $Dilaton(Q) = Q'$. In the diagram below, $|OP| = 3$ cm and $|OQ| = 4$ cm, as shown.



2. If the scale factor is $r = 3$, what is the length of segment OP'?

3. Use the definition of dilation to show that your answer to Exercise 2 is correct.

4. If the scale factor is $r = 3$, what is the length of segment OQ'?

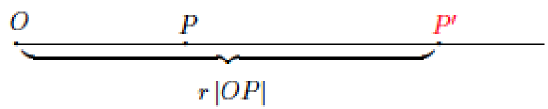
5. Use the definition of dilation to show that your answer to Exercise 4 is correct.

6. If you know that $|OP| = 3$, $|OP'| = 9$, how could you use that information to determine the scale factor?

Lesson Summary

Definition: For a positive number r, a *dilation with center O and scale factor r* is the transformation of the plane that maps O to itself, and maps each remaining point P of the plane to its image P'on the ray $\overrightarrow{OP}$ so that $|OP'| = r|OP|$. That is, it is the transformation that assigns to each point P of the plane a point $Dilation(P)$ so that

1. $Dilation(O) = O$ (i.e., a dilation does not move the center of dilation).

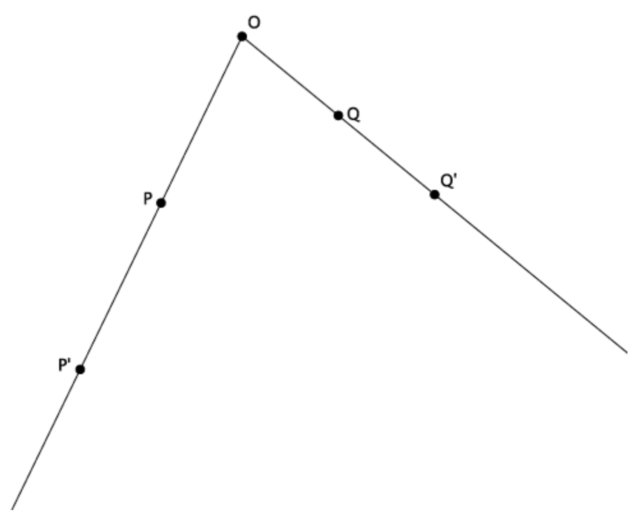


2. If $P \neq O$, then the point $Dilation(P)$ (to be denoted more simply by P') is the point on the ray $\overrightarrow{OP}$ so that $|OP'| = r|OP|$.

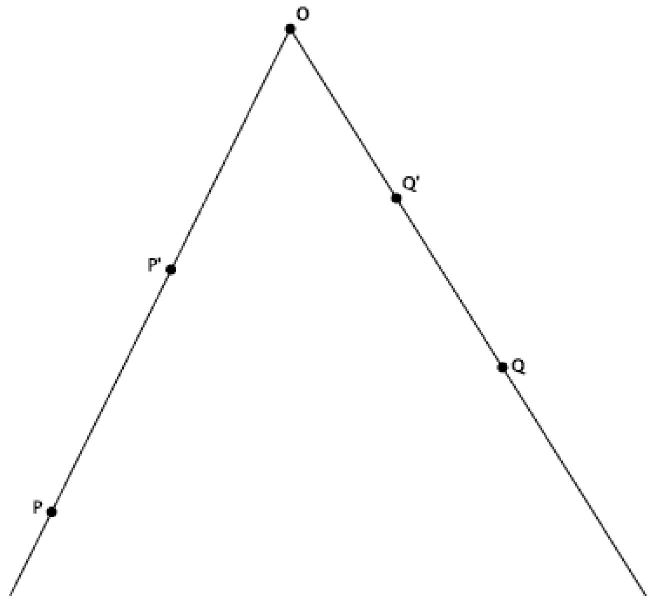
In other words, a dilation is a rule that moves each point P along the ray emanating from the center O to a new point P' on that ray such that the distance $|OP'|$ is r times the distance $|OP|$.

Problem Set

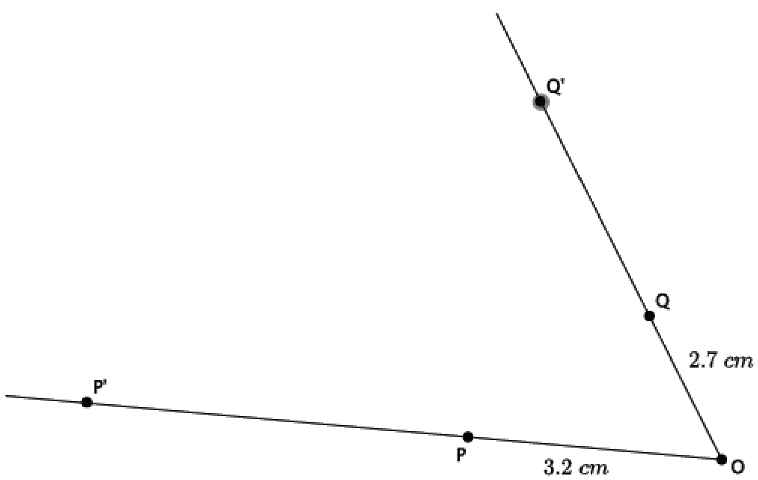
1. Let there be a dilation from center O. Then, $Dilation(P) = P'$ and $Dilation(Q) = Q'$. Examine the drawing below. What can you determine about the scale factor of the dilation?



2. Let there be a dilation from center O. Then, $Dilation(P) = P'$, and $Dilation(Q) = Q'$. Examine the drawing below. What can you determine about the scale factor of the dilation?



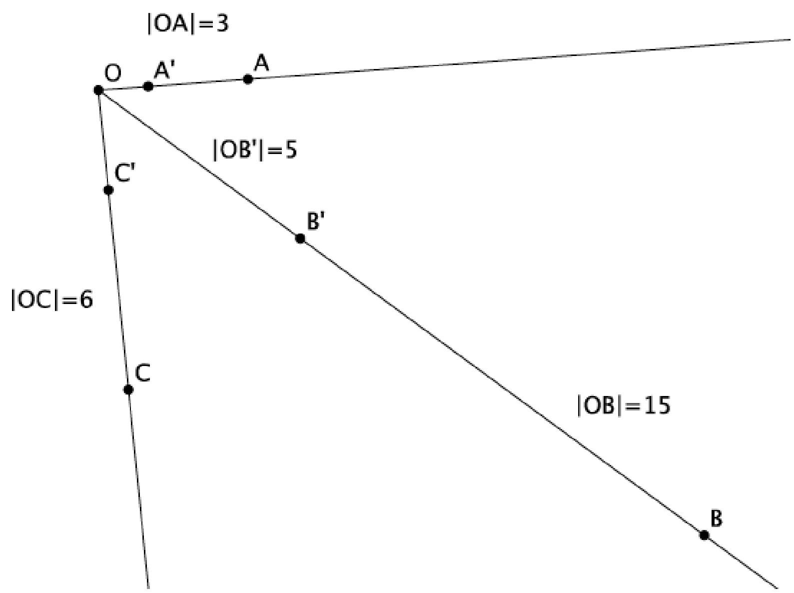
3. Let there be a dilation from center O with a scale factor $r = 4$. Then, $Dilation(P) = P'$ and $Dilation(Q) = Q'$. $|OP| = 3.2 \text{ cm}$, and $|OQ| = 2.7 \text{ cm}$, as shown. Use the drawing below to answer parts (a) and (b). The drawing is not to scale.



a. Use the definition of dilation to determine $|OP'|$.

b. Use the definition of dilation to determine $|OQ'|$.

4. Let there be a dilation from center O with a scale factor r. Then, $Dilation(A) = A'$, $Dilation(B) = B'$, and $Dilation(C) = C'$. $|OA| = 3$, $|OB| = 15$, $|OC| = 6$, and $|OB'| = 5$, as shown. Use the drawing below to answer parts (a)–(c).



a. Using the definition of dilation with lengths OB and OB', determine the scale factor of the dilation.

b. Use the definition of dilation to determine $|OA'|$.

c. Use the definition of dilation to determine $|OC'|$.