Lesson 11: Efficacy of Scientific Notation

Student Outcomes

- Students continue to practice working with very small and very large numbers expressed in scientific notation.
- Students read, write, and perform operations on numbers expressed in scientific notation.

Lesson Notes

Powers of Ten, a video that demonstrates positive and negative powers of 10, is available online. The video should pique students' interest in why exponential notation is necessary. A link to the video is provided below (9 minutes).

http://www.youtube.com/watch?v=0fKBhvDjuy0

Classwork

Exercises 1–2 (3 minutes)

Exercise 1

The mass of a proton is

$$0.000\,000\,000\,000\,000\,000\,000\,000\,001\,672\,622 \text{ kg.}$$

In scientific notation it is

1.672622×10^{-27} kg.

Exercise 2

The mass of an electron is

$$0.000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,910\,938\,291 \text{ kg.}$$

In scientific notation it is

$9.10938291 \times 10^{-31}$ kg.

Discussion (3 minutes)

MP.6

We continue to discuss why it is important to express numbers using scientific notation.

Consider the mass of the proton

$$0.000\,000\,000\,000\,000\,000\,000\,000\,001\,672\,622 \text{ kg.}$$

It is more informative to write it in scientific notation

$$1.672622 \times 10^{-27} \text{ kg.}$$

The exponent -27 is used because the first nonzero digit (i.e., 1) of this number occurs in the 27^{th} digit after the decimal point.

MP.6

Similarly, the mass of the electron is

$$0.000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,910\,938\,291 \text{ kg}.$$

It is much easier to read this number in scientific notation

$$9.10938291 \times 10^{-31} \text{ kg}.$$

In this case, the exponent -31 is used because the first nonzero digit (i.e., 9) of this number occurs in the 31^{st} digit to the right of the decimal point.

Scaffolding:

Some time should be spent making sense of these statements with students. This can be accomplished by, for example, providing a series of simpler numbers (e.g., $0.1, 0.01, 0.001$) and demonstrating that when they are expressed in exponential notation, $10^{-1}, 10^{-2}, 10^{-3}$, that the exponent tells that the first nonzero digit occurs in the first, second, and third digit to the right of the decimal point, respectively.

Exercise 3 (3 minutes)

Before students write the ratio that compares the mass of a proton to the mass of an electron, ask them whether they would rather write the ratio in standard (i.e., decimal) or scientific notation. If necessary, help them understand why scientific notation is more advantageous.

Exercise 3

Write the ratio that compares the mass of a proton to the mass of an electron.

Ratio: $(1.672622 \times 10^{-27}) : (9.10938291 \times 10^{-31})$

Discussion (20 minutes)

This discussion includes Example 1, Exercise 4, and Example 2.

Example 1

The advantage of the scientific notation becomes even more pronounced when we have to compute how many times heavier a proton is than an electron. Instead of writing the value of the ratio, r, as

$$r = \frac{0.000\,000\,000\,000\,000\,000\,000\,000\,001\,672\,622}{0.000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,910\,938\,291}$$

we express it as

$$r = \frac{1.672622 \times 10^{-27}}{9.10938291 \times 10^{-31}}.$$

MP.2

- Should we eliminate the power of 10 in the numerator or denominator? Explain.
  - *Using the theorem on generalized equivalent fractions, we can eliminate the negative power of* 10 *in the numerator and denominator to see what we are doing more clearly. Anticipating that* $10^{-31} \times 10^{31} = 1$, *we can multiply the numerator and denominator of the (complex) fraction by* 10^{31}

Scaffolding:

When students ask why we eliminated the negative power of 10 in the denominator (instead of the numerator), explain that positive powers of 10 are easier to interpret.

$$r = \frac{1.672622 \times 10^{-27}}{9.10938291 \times 10^{-31}} \times \frac{10^{31}}{10^{31}}.$$

Using the first law of exponents (10) presented in Lesson 5, we get

$$r = \frac{1.672622 \times 10^{4}}{9.10938291} = \frac{1.672622}{9.10938291} \times 10^{4}.$$

MP.2 Note that since we are using scientific notation, we can interpret an approximate value of r right away. For example, we see

$$\frac{1.672622}{9.10938291} \approx \frac{1.7}{9.1} = \frac{17}{91} \approx \frac{1}{5}$$

so that r is approximately $\frac{1}{5} \times 10{,}000$, which is 2,000. Thus, we expect a proton to be about two thousand times heavier than an electron.

Exercise 4

Students find a more precise answer for Example 1. Allow students to use a calculator to divide 1.672 622 by 9.109 382 91. When they finish, have students compare the approximate answer (2,000) to their more precise answer (1,836).

Exercise 4

Compute how many times heavier a proton is than an electron (i.e., find the value of the ratio). Round your final answer to the nearest one.

Let $r =$ the value of the ratio, then:

$$\begin{aligned}
r &= \frac{1.672622 \times 10^{-27}}{9.10938291 \times 10^{-31}}\\
&= \frac{1.672622 \times 10^{-27} \times 10^{31}}{9.10938291 \times 10^{-31} \times 10^{31}}\\
&= \frac{1.672622 \times 10^{4}}{9.10938291}\\
&= \frac{1.672622}{9.10938291} \times 10^{4}\\
&= \frac{1.672622 \times 10^{8}}{9.10938291 \times 10^{8}} \times 10^{4}\\
&= \frac{167{,}262{,}200}{910{,}938{,}291} \times 10^{4}\\
&= 0.183615291675 \times 10^{4}\\
&= 1836.15291675\\
&\approx 1836
\end{aligned}$$

Example 2

As of March 23, 2013, the U.S. national debt was $\$16{,}755{,}133{,}009{,}522$ (rounded to the nearest dollar). According to the 2012 U.S. census, there are about $313{,}914{,}040$ American citizens. What is each citizen's approximate share of the debt?

- How precise should we make our answer? Do we want to know the exact amount, to the nearest dollar, or is a less precise answer alright?
  - *The most precise answer uses the exact numbers listed in the problem. The more the numbers are rounded, the precision of the answer decreases. We should aim for the most precise answer when necessary, but the following problem does not require it since we are finding the "approximate share of the debt."*

Let's round off the debt to the nearest *billion* (10^9). It is $\$16{,}755{,}000{,}000{,}000$, which is 1.6755×10^{13} dollars. Let's also round off the population to the nearest *million* (10^6), making it $314{,}000{,}000$, which is 3.14×10^8. Therefore, using the product formula and equation (13) from Lesson 5, we see that each citizen's share of the debt, in dollars, is

$$\begin{aligned}\frac{1.6755 \times 10^{13}}{3.14 \times 10^8} &= \frac{1.6755}{3.14} \times \frac{10^{13}}{10^8}\\ &= \frac{1.6755}{3.14} \times 10^5.\end{aligned}$$

Once again, we note the advantages of computing numbers expressed in scientific notation. Immediately, we can approximate the answer, about half of 10^5, or a hundred thousand dollars, (i.e., about $\$50{,}000$), because

$$\frac{1.6755}{3.14} \approx \frac{1.7}{3.1} = \frac{17}{31} \approx \frac{1}{2}.$$

More precisely, with the help of a calculator,

$$\frac{1.6755}{3.14} = \frac{16755}{31410} = 0.533598\ldots \approx 0.5336.$$

Therefore, each citizen's share of the U.S. national debt is about $\$53{,}360$.

Example 2

The U.S. national debt as of March 23, 2013, rounded to the nearest dollar, is $\$16{,}755{,}133{,}009{,}522$. According to the 2012 U.S. census, there are about $313{,}914{,}040$ U.S. citizens. What is each citizen's approximate share of the debt?

$$\begin{aligned}\frac{1.6755 \times 10^{13}}{3.14 \times 10^8} &= \frac{1.6755}{3.14} \times \frac{10^{13}}{10^8}\\ &= \frac{1.6755}{3.14} \times 10^5\\ &= 0.533598\ldots \times 10^5\\ &\approx 0.5336 \times 10^5\\ &= 53360\end{aligned}$$

Each U.S. citizen's share of the national debt is about **$\$53{,}360$.**

Exercises 5–6 (8 minutes)

Students work on Exercises 5 and 6 independently.

Exercise 5

The geographic area of California is $163,696$ sq. mi., and the geographic area of the U.S. is $3,794,101$ sq. mi. Let's round off these figures to 1.637×10^5 and 3.794×10^6. In terms of area, roughly estimate how many Californias would make up one U.S. Then compute the answer to the nearest ones.

$$\begin{aligned}\frac{3.794 \times 10^6}{1.637 \times 10^5} &= \frac{3.794}{1.637} \times \frac{10^6}{10^5} \\ &= \frac{3.794}{1.637} \times 10 \\ &= 2.3176\ldots \times 10 \\ &\approx 2.318 \times 10 \\ &= 23.18\end{aligned}$$

It would take about 23 Californias to make up one U.S.

Exercise 6

The average distance from Earth to the moon is about 3.84×10^5 km, and the distance from Earth to Mars is approximately 9.24×10^7 km in year 2014. On this simplistic level, how much farther is traveling from Earth to Mars than from Earth to the moon?

$$\begin{aligned}9.24 \times 10^7 - 3.84 \times 10^5 &= 924 \times 10^5 - 3.84 \times 10^5 \\ &= (924 - 3.84) \times 10^5 \\ &= 920.16 \times 10^5 \\ &= 92\,016\,000\end{aligned}$$

It is $92,016,000$ km further to travel from Earth to Mars than from Earth to the moon.

Closing (3 minutes)

Summarize, or have students summarize, the lesson.

- We can read, write, and operate with numbers expressed in scientific notation.

Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 11: Efficacy of the Scientific Notation

Exit Ticket

1. Two of the largest mammals on earth are the blue whale and the African elephant. An adult male blue whale weighs about 170 tonnes or long tons. (1 tonne $= 1000$ kg)

   Show that the weight of an adult blue whale is 1.7×10^5 kg.

2. An adult male African elephant weighs about 9.07×10^3 kg.

   Compute how many times heavier an adult male blue whale is than an adult male African elephant (i.e., find the value of the ratio). Round your final answer to the nearest one.

Exit Ticket Sample Solutions

1. Two of the largest mammals on earth are the blue whale and the elephant. An adult male blue whale weighs about 170 tonnes or long tons. (1 tonne $= 1000$ kg)

   Show that the weight of an adult blue whale is 1.7×10^5 kg.

   Let x (or any other symbol) represent the number of kilograms an adult blue whale weighs.

$$170 \times 1000 = x$$
$$1.7 \times 10^5 = x$$

2. An adult male elephant weighs about 9.07×10^3 kg.

   Compute how many times heavier an adult male blue whale is than an adult male elephant (i.e., find the value of the ratio). Round your final answer to the nearest one.

   Let r be the value of the ratio.

$$\begin{aligned} r &= \frac{1.7 \times 10^5}{9.07 \times 10^3} \\ &= \frac{1.7}{9.07} \times 10^2 \\ &= 0.18743 \times 10^2 \\ &= 18.743 \\ &\approx 19 \end{aligned}$$

   The blue whale is 19 times heavier than the elephant.

Problem Set Sample Solutions

1. There are approximately 7.5×10^{18} grains of sand on Earth. There are approximately 7×10^{27} atoms in an average human body. Are there more grains of sand on Earth or atoms in an average human body? How do you know?

   There are more atoms in the average human body. When comparing the order of magnitude of each number, $27 > 18$; therefore, $7 \times 10^{27} > 7.5 \times 10^{18}$.

2. About how many times more atoms are in a human body compared to grains of sand on Earth?

$$\begin{aligned} \frac{7 \times 10^{27}}{7.5 \times 10^{18}} &= \frac{7}{7.5} \times \frac{10^{27}}{10^{18}} \\ &\approx 1 \times 10^{27-18} \\ &\approx 1 \times 10^9 \\ &\approx 10^9 \end{aligned}$$

   There are about $1,000,000,000$ times more atoms in the human body compared to grains of sand on Earth.

3. Suppose the geographic areas of California and the U.S. are 1.637×10^5 and 3.794×10^6 sq. mi., respectively. California's population (as of 2012) is approximately 3.804×10^7 people. If population were proportional to area, what would be the U.S. population?

We already know from Exercise 5 that it would take about 23 Californias to make up one U.S. Then the population of the U.S. would be 23 times the population of California, which is

$$\begin{aligned} 23 \times 3.804 \times 10^7 &= 87.492 \times 10^7 \\ &= 8.7492 \times 10^8 \\ &= 874{,}920{,}000. \end{aligned}$$

4. The actual population of the U.S. (as of 2012) is approximately 3.14×10^8. How does the population density of California (i.e., the number of people per square mile) compare with the population density of the U.S.?

Population density of California per square mile:

$$\begin{aligned} \frac{3.804 \times 10^7}{1.637 \times 10^5} &= \frac{3.804}{1.637} \times \frac{10^7}{10^5} \\ &= 2.32376\ldots \times 10^2 \\ &\approx 2.32 \times 10^2 \\ &= 232 \end{aligned}$$

Population density of the U.S. per square mile:

$$\begin{aligned} \frac{3.14 \times 10^8}{3.794 \times 10^6} &= \frac{3.14}{3.794} \times \frac{10^8}{10^6} \\ &= 0.8276\ldots \times 10^2 \\ &\approx 0.83 \times 10^2 \\ &= 83 \end{aligned}$$

Population density of California compared to the population density of the U.S.:

$$\begin{aligned} \frac{232}{83} &= 2.7951\ldots \\ &\approx 2.8 \end{aligned}$$

California is about 3 times as dense as the U.S. in terms of population.