Lesson 10: Operations with Numbers in Scientific Notation

Student Outcomes

- Students practice operations with numbers expressed in scientific notation and standard notation.

Classwork

Examples 1–2 (8 minutes)

Example 1: The world population is about 7 billion. There are 4.6×10^7 ants for every human on the planet. About how many ants are there in the world?

First, write 7 billion in scientific notation: (7×10^9).

To find the number of ants in the world, we need to multiply the world population by the known number of ants for each person: $(7 \times 10^9)(4.6 \times 10^7)$.

$(7 \times 10^9)(4.6 \times 10^7) = (7 \times 4.6)(10^9 \times 10^7)$	By repeated use of the associative and commutative properties
$= 32.2 \times 10^{16}$	By the first law of exponents
$= 3.22 \times 10 \times 10^{16}$	
$= 3.22 \times 10^{17}$	By the first law of exponents

There are about 3.22×10^{17} *ants in the world!*

Example 2: A certain social media company processes about 990 billion *likes* per year. If the company has approximately 8.9×10^8 users of the social media, about how many *likes* is each user responsible for per year? Write your answer in scientific and standard notation.

First, write 990 billion in scientific notation: 9.9×10^{11}.

To find the number of *likes* per person, divide the total number of *likes* by the total number of users: $\frac{9.9 \times 10^{11}}{8.9 \times 10^8}$

$\frac{9.9 \times 10^{11}}{8.9 \times 10^8} = \frac{9.9}{8.9} \times \frac{10^{11}}{10^8}$	By the product formula
$= 1.11235\ldots \times 10^3$	By the first law of exponents
$\approx 1.1 \times 10^3$	
≈ 1100	

Each user is responsible for about 1.1×10^3*, or* 1,100*, likes per year.*

Exercises 1– 2 (10 minutes)

Have students complete Exercises 1 and 2 independently.

Exercise 1

The speed of light is $300,000,000$ meters per second. The sun is approximately 1.5×10^{11} meters from Earth. How many seconds does it take for sunlight to reach Earth?

$$300\,000\,000 = 3 \times 10^8$$
$$\frac{1.5 \times 10^{11}}{3 \times 10^8} = \frac{1.5}{3} \times \frac{10^{11}}{10^8}$$
$$= 0.5 \times 10^3$$
$$= 0.5 \times 10 \times 10^2$$
$$= 5 \times 10^2$$

It takes 500 seconds for sunlight to reach Earth.

Exercise 2

The mass of the moon is about 7.3×10^{22} kg. It would take approximately $26,000,000$ moons to equal the mass of the sun. Determine the mass of the sun.

$$26\,000\,000 = 2.6 \times 10^7$$
$$(2.6 \times 10^7)(7.3 \times 10^{22}) = (2.6 \times 7.3)(10^7 \times 10^{22})$$
$$= 18.98 \times 10^{29}$$
$$= 1.898 \times 10 \times 10^{29}$$
$$= 1.898 \times 10^{30}$$

The mass of the sun is 1.898×10^{30} kg.

Example 3 (8 minutes)

In 2010, Americans generated 2.5×10^8 tons of garbage. There are about 2,000 landfills in the United States. Assuming that each landfill is the same size and that trash is divided equally among them, determine how many tons of garbage were sent to each landfill in 2010.

First, write 2,000 in scientific notation: 2×10^3.

To find the number of tons of garbage sent to each landfill, divide the total weight of the garbage by the number of landfills: $\frac{2.5 \times 10^8}{2 \times 10^3}$.

$$\frac{2.5 \times 10^8}{2 \times 10^3} = \frac{2.5}{2} \times \frac{10^8}{10^3} \quad \text{By the product formula}$$
$$= 1.25 \times 10^5 \quad \text{By the first law of exponents}$$

Each landfill received 1.25×10^5 tons of garbage in 2010.

Actually, not all garbage went to landfills. Some of it was recycled and composted. The amount of recycled and composted material accounted for about 85 million tons of the 2.5×10^8 tons of garbage. Given this new information, how much garbage was actually sent to each landfill?

First, write 85 million in scientific notation: 8.5×10^7.

Next, subtract the amount of recycled and composted material from the garbage: $2.5 \times 10^8 - 8.5 \times 10^7$. To subtract, we must give each number the same order of magnitude and then use the distributive property.

$$\begin{aligned} 2.5 \times 10^8 - 8.5 \times 10^7 &= (2.5 \times 10) \times 10^7 - 8.5 \times 10^7 && \text{By the first law of exponents} \\ &= (2.5 \times 10) - 8.5)) \times 10^7 && \text{By the distributive property} \\ &= (25 - 8.5) \times 10^7 \\ &= 16.5 \times 10^7 \\ &= 1.65 \times 10 \times 10^7 \\ &= 1.65 \times 10^8 && \text{By the first law of exponents} \end{aligned}$$

Now, divide the new amount of garbage by the number of landfills: $\frac{1.65 \times 10^8}{2 \times 10^3}$.

$$\begin{aligned} \frac{1.65 \times 10^8}{2 \times 10^3} &= \frac{1.65}{2} \times \frac{10^8}{10^3} && \text{By the product formula} \\ &= 0.825 \times 10^5 && \text{By the first law of exponents} \\ &= 0.825 \times 10 \times 10^4 && \text{By the first law of exponents} \\ &= 8.25 \times 10^4 \end{aligned}$$

Each landfill actually received 8.25×10^4 *tons of garbage in* 2010.

Exercises 3–5 (10 minutes)

Have students complete Exercises 3–5 independently.

Exercise 3

The mass of Earth is 5.9×10^{24} kg. The mass of Pluto is $13,000,000,000,000,000,000,000$ kg. Compared to Pluto, how much greater is Earth's mass than Pluto's mass?

$$13\,000\,000\,000\,000\,000\,000\,000 = 1.3 \times 10^{22}$$

$$\begin{aligned} 5.9 \times 10^{24} - 1.3 \times 10^{22} &= (5.9 \times 10^2) \times 10^{22} - 1.3 \times 10^{22} \\ &= (590 - 1.3) \times 10^{22} \\ &= 588.7 \times 10^{22} \\ &= 5.887 \times 10^2 \times 10^{22} \\ &= 5.887 \times 10^{24} \end{aligned}$$

The mass of Earth is 5.887×10^{24} kg greater than the mass of Pluto.

Exercise 4

Using the information in Exercises 2 and 3, find the combined mass of the moon, Earth, and Pluto.

$$\begin{aligned}7.3\times10^{22}+1.3\times10^{22}+5.9\times10^{24} &= (7.3\times10^{22}+1.3\times10^{22})+5.9\times10^{24}\\ &= 8.6\times10^{22}+5.9\times10^{24}\\ &= (8.6+590)\times10^{22}\\ &= 598.6\times10^{22}\\ &= 5.986\times10^{2}\times10^{22}\\ &= 5.986\times10^{24}\end{aligned}$$

The combined mass of the moon, Earth, and Pluto is 5.986×10^{24} kg.

Exercise 5

How many combined moon, Earth, and Pluto masses (i.e., the answer to Exercise 4) are needed to equal the mass of the sun (i.e., the answer to Exercise 2)?

$$\begin{aligned}\frac{1.898\times10^{30}}{5.986\times10^{24}} &= \frac{1.898}{5.986}\times\frac{10^{30}}{10^{24}}\\ &= 0.3170\ldots\times10^{6}\\ &\approx 0.32\times10^{6}\\ &= 0.32\times10\times10^{5}\\ &= 3.2\times10^{5}\end{aligned}$$

It would take 3.2×10^{5} combined masses of the moon, Earth, and Pluto to equal the mass of the sun.

Closing (4 minutes)

Summarize, or have students summarize, the lesson.

- We can perform all operations for numbers expressed in scientific notation or standard notation.

Exit Ticket (5 minutes)

Name ____ Date ____________________

Lesson 10: Operations with Numbers in Scientific Notation

Exit Ticket

1. The speed of light is 3×10^8 meters per second. The sun is approximately $230{,}000{,}000{,}000$ meters from Mars. How many seconds does it take for sunlight to reach Mars?

2. If the sun is approximately 1.5×10^{11} meters from Earth, what is the approximate distance from Earth to Mars?

Exit Ticket Sample Solutions

1. **The speed of light is 3×10^8 meters per second. The sun is approximately $230,000,000,000$ meters from Mars. How many seconds does it take for sunlight to reach Mars?**

$$230\,000\,000\,000 = 2.3 \times 10^{11}$$
$$\frac{2.3 \times 10^{11}}{3 \times 10^8} = \frac{2.3}{3} \times \frac{10^{11}}{10^8}$$
$$= 0.7666\ldots \times 10^3$$
$$\approx 0.77 \times 10 \times 10^2$$
$$\approx 7.7 \times 10^2$$

It takes approximately 770 seconds for sunlight to reach Mars.

2. **If the sun is approximately 1.5×10^{11} meters from Earth, what is the approximate distance from Earth to Mars?**

$$(2.3 \times 10^{11}) - (1.5 \times 10^{11}) = (2.3 - 1.5) \times 10^{11}$$
$$= 0.8 \times 10^{11}$$
$$= 0.8 \times 10 \times 10^{10}$$
$$= 8 \times 10^{10}$$

The distance from Earth to Mars is 8×10^{10} meters.

Problem Set Sample Solutions

Have students practice operations with numbers written in scientific notation and standard notation.

1. **The sun produces 3.8×10^{27} joules of energy per second. How much energy is produced in a year? (Note: a year is approximately $31,000,000$ seconds).**

$$31\,000\,000 = 3.1 \times 10^7$$
$$(3.8 \times 10^{27})(3.1 \times 10^7) = (3.8 \times 3.1)(10^{27} \times 10^7)$$
$$= 11.78 \times 10^{34}$$
$$= 1.178 \times 10 \times 10^{34}$$
$$= 1.178 \times 10^{35}$$

The sun produces 1.178×10^{35} joules of energy in a year.

2. On average, Mercury is about $57,000,000$ km from the sun, whereas Neptune is about 4.5×10^9 km from the sun. What is the difference between Mercury's and Neptune's distances from the sun?

$$57\,000\,000 = 5.7 \times 10^7$$
$$\begin{aligned}4.5 \times 10^9 - 5.7 \times 10^7 &= (4.5 \times 10^2) \times 10^7 - 5.7 \times 10^7 \\ &= 450 \times 10^7 - 5.7 \times 10^7 \\ &= (450 - 5.7) \times 10^7 \\ &= 444.3 \times 10^7 \\ &= 4.443 \times 10^2 \times 10^7 \\ &= 4.443 \times 10^9\end{aligned}$$

The difference in the distance of Mercury and Neptune from the sun is 4.443×10^9 km.

3. The mass of Earth is approximately 5.9×10^{24} kg, and the mass of Venus is approximately 4.9×10^{24} kg.

a. Find their combined mass.

$$\begin{aligned}5.9 \times 10^{24} + 4.9 \times 10^{24} &= (5.9 + 4.9) \times 10^{24} \\ &= 10.8 \times 10^{24} \\ &= 1.08 \times 10 \times 10^{24} \\ &= 1.08 \times 10^{25}\end{aligned}$$

The combined mass of Earth and Venus is 1.08×10^{25} kg.

b. Given that the mass of the sun is approximately 1.9×10^{30} kg, how many Venuses and Earths would it take to equal the mass of the sun?

$$\begin{aligned}\frac{1.9 \times 10^{30}}{1.08 \times 10^{25}} &= \frac{1.9}{1.08} \times \frac{10^{30}}{10^{25}} \\ &= 1.75925\ldots \times 10^5 \\ &\approx 1.8 \times 10^5\end{aligned}$$

It would take approximately 1.8×10^5 *Venuses and Earths to equal the mass of the sun.*