



Table of Contents¹

Linear Functions

Module Overview	2
Topic A: Linear Functions (8.F.B.4, 8.F.B.5)	7
Lesson 1: Modeling Linear Relationships	8
Lesson 2: Interpreting Rate of Change and Initial Value	20
Lesson 3: Representations of a Line	28
Lessons 4–5: Increasing and Decreasing Functions	39
Topic B: Bivariate Numerical Data (8.SP.A.1, 8.SP.A.2)	67
Lesson 6: Scatter Plots	68
Lesson 7: Patterns in Scatter Plots	79
Lesson 8: Informally Fitting a Line	95
Lesson 9: Determining the Equation of a Line Fit to Data	107
Mid-Module Assessment and Rubric	119
<i>Topics A through B (assessment 1 day, return 1 day, remediation or further applications 1 day)</i>	
Topic C: Linear and Nonlinear Models (8.SP.A.1, 8.SP.A.2, 8.SP.A.3)	129
Lesson 10: Linear Models	130
Lesson 11: Using Linear Models in a Data Context	142
Lesson 12: Nonlinear Models in a Data Context (Optional)	153
Topic D: Bivariate Categorical Data (8.SP.A.4)	167
Lesson 13: Summarizing Bivariate Categorical Data in a Two-Way Table	168
Lesson 14: Association Between Categorical Variables	180
End-of-Module Assessment and Rubric	190
<i>Topics A through D (assessment 1 day, return 1 day, remediation or further applications 1 day)</i>	

¹Each lesson is ONE day, and ONE day is considered a 45-minute period.

Grade 8 • Module 6

Linear Functions

OVERVIEW

In Grades 6 and 7, students worked with data involving a single variable. This module introduces students to bivariate data. Students are introduced to a function as a rule that assigns exactly one value to each input. In this module, students use their understanding of functions to model the relationships of bivariate data. This module is important in setting a foundation for students' work in Algebra I.

Topic A examines the relationship between two variables using linear functions (**8.F.B.4**). Linear functions are connected to a context using the initial value and slope as a rate of change to interpret the context. Students represent linear functions by using tables and graphs and by specifying rate of change and initial value. Slope is also interpreted as an indication of whether the function is increasing or decreasing and as an indication of the steepness of the graph of the linear function (**8.F.B.5**). Nonlinear functions are explored by examining nonlinear graphs and verbal descriptions of nonlinear behavior.

In Topic B, students use linear functions to model the relationship between two quantitative variables as students move to the domain of statistics and probability. Students make scatter plots based on data. They also examine the patterns of their scatter plots or given scatter plots. Students assess the fit of a linear model by judging the closeness of the data points to the line (**8.SP.A.1**, **8.SP.A.2**).

In Topic C, students use linear and nonlinear models to answer questions in context (**8.SP.A.1**, **8.SP.A.2**). They interpret the rate of change and the initial value in context (**8.SP.A.3**). They use the equation of a linear function and its graph to make predictions. Students also examine graphs of nonlinear functions and use nonlinear functions to model relationships that are nonlinear. Students gain experience with the mathematical practice of “modeling with mathematics” (MP.4).

In Topic D, students examine bivariate categorical data by using two-way tables to determine relative frequencies. They use the relative frequencies calculated from tables to informally assess possible associations between two categorical variables (**8.SP.A.4**).

Focus Standards

Use functions to model relationships between quantities.

- 8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x,y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

- 8.F.B.5** Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Investigate patterns of association in bivariate data.²

- 8.SP.A.1** Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- 8.SP.A.2** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
- 8.SP.A.3** Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
- 8.SP.A.4** Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Foundational Standards

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

- 7.EE.B.4** Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Define, evaluate, and compare functions.

- 8.F.B.1** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.³
- 8.F.B.2** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

²8.SP standards are used as applications to the work done with 8.F standards.

³Function notation is not required in Grade 8.

- 8.F.B.3** Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Focus Standards for Mathematical Practice

- MP.2 Reason abstractly and quantitatively.** Students reason quantitatively by symbolically representing the verbal description of a relationship between two bivariate variables. They attend to the meaning of data based on the context of problems and the possible linear or nonlinear functions that explain the relationships of the variables.
- MP.4 Model with mathematics.** Students model relationships between variables using linear and nonlinear functions. They interpret models in the context of the data and reflect on whether or not the models make sense based on slopes, initial values, or the fit to the data.
- MP.6 Attend to precision.** Students evaluate functions to model a relationship between numerical variables. They evaluate the function by assessing the closeness of the data points to the line. They use care in interpreting the slope and the y -intercept in linear functions.
- MP.7 Look for and make use of structure.** Students identify pattern or structure in scatter plots. They fit lines to data displayed in a scatter plot and determine the equations of lines based on points or the slope and initial value.

Terminology

New or Recently Introduced Terms

- **Association (description)** (An *association* is a relationship between the two variables of a bivariate data set.
The relationship is often expressed in terms of relative frequencies (described using two-way tables of the two domains of variables of the data set) or numerical relationships that can be modeled by functions (most often as linear relationships between the two domains of the two variables for the data set).)
- **Bivariate Data Set (description)** (A *bivariate data set* is an ordered list of ordered pairs of data values (called *data points*).
Data sets and bivariate data sets are both called *data sets*. Data values can be either numerical or categorical. If both are numerical, then the data set is called a *numerical bivariate data set*.)
- **Column Relative Frequency** (In a two-way table, a *column relative frequency* is a cell frequency divided by the column total for that cell.)
- **Row Relative Frequency** (In a two-way table, a *row relative frequency* is the number given by dividing the cell frequency by the row total for that cell.)
- **Scatter Plot** (A *scatter plot* is a graph of a numerical bivariate data set.)

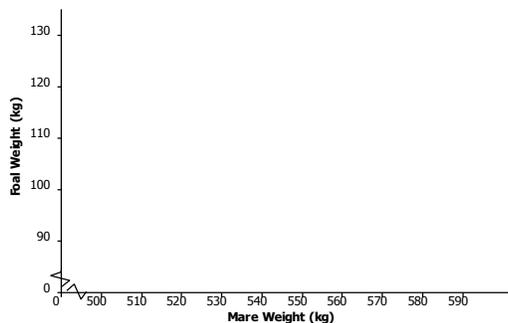
- **Two-Way Frequency Table (description)** (A *two-way frequency table* is a rectangular table used to summarize data on two categorical variables of a bivariate data set. The rows of the table correspond to the possible categories for one of the variables, and the columns correspond to the possible categories for the other variable. Entries in the cells of the table indicate the number of times that a particular category combination occurs in the data set; the value is the frequency for that combination.)
- **Variable (description)** (A *variable* is a symbol (such as a letter) that is a placeholder for a data value from a specified set of data values. The specified set of data values is called the *domain* of the variable.)

Familiar Terms and Symbols⁴

- Categorical variable
- Intercept or initial value
- Numerical variable
- Slope

Suggested Tools and Representations

- Graphing calculator
- Scatter Plot
- Two-way frequency tables



Scatter Plot

	Curfew	No Curfew	Total
Assigned Chores	25	10	35
Not Assigned Chores	8	7	15
Total	33	17	50

Two-Way Frequency Table

⁴These are terms and symbols students have seen previously.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	8.F.B.4, 8.F.B.5, 8.SP.A.1, 8.SP.A.2
End-of-Module Assessment Task	After Topic D	Constructed response with rubric	8.F.B.4, 8.F.B.5, 8.SP.A.1, 8.SP.A.2, 8.SP.A.3, 8.SP.A.4