

Lesson 39: Factoring Extended to the Complex Realm

Classwork

Opening Exercise

Rewrite each expression as a polynomial in standard form.

a. $(x + i)(x - i)$

b. $(x + 5i)(x - 5i)$

c. $(x - (2 + i))(x - (2 - i))$

Exercises 1–4

Factor the following polynomial expressions into products of linear terms.

1. $x^2 + 9$

2. $x^2 + 5$

4. Write a polynomial P with the lowest possible degree that has the given solutions. Explain how you generated each answer.

a. $-2, 3, -4i, 4i$

b. $-1, 3i$

c. $0, 2, 1 + i, 1 - i$

d. $\sqrt{2}, -\sqrt{2}, 3, 1 + 2i$

e. $2i, 3 - i$

Lesson Summary

- Polynomial equations with real coefficients can have real or complex solutions or they can have both.
- If a complex number is a solution to a polynomial equation, then its conjugate is also a solution.
- Real solutions to polynomial equations correspond to the x -intercepts of the associated graph, but complex solutions do not.

Problem Set

1. Rewrite each expression in standard form.
 - a. $(x + 3i)(x - 3i)$
 - b. $(x - a + bi)(x - (a + bi))$
 - c. $(x + 2i)(x - i)(x + i)(x - 2i)$
 - d. $(x + i)^2 \cdot (x - i)^2$
2. Suppose in Problem 1 that you had no access to paper, writing utensils, or technology. How do you know that the expressions in parts (a)–(d) are polynomials with real coefficients?
3. Write a polynomial equation of degree 4 in standard form that has the solutions i , $-i$, 1 , -1 .
4. Explain the difference between x -intercepts and solutions to an equation. Give an example of a polynomial with real coefficients that has twice as many solutions as x -intercepts. Write it in standard form.
5. Find the solutions to $x^4 - 5x^2 - 36 = 0$ and the x -intercepts of the graph of $y = x^4 - 5x^2 - 36$.
6. Find the solutions to $2x^4 - 24x^2 + 40 = 0$ and the x -intercepts of the graph of $y = 2x^4 - 24x^2 + 40$.
7. Find the solutions to $x^4 - 64 = 0$ and the x -intercepts of the graph of $y = x^4 - 64$.
8. Use the fact that $x^4 + 64 = (x^2 - 4x + 8)(x^2 + 4x + 8)$ to explain how you know that the graph of $y = x^4 + 64$ has no x -intercepts. You need not find the solutions.